GRADE 5

Mathematics Assessment Anchors and Eligible content: Aligned to Pennsylvania Common Core Standards

The Assessment Anchors, as defined by the Eligible Content, are organized into cohesive blueprints, each

structured with a common labeling system that can be read like an outline. This framework is organized first by

Reporting Category, then by Assessment Anchor, followed by Anchor Descriptor, and then finally, at the greatest level of detail, by an Eligible Content statement. The common format of this outline is followed across the PSSA.

Here is a description of each level in the labeling system for the PSSA:

**Reporting Category**

The Assessment Anchors are organized into four classifications, as listed below.

•A = Numbers and Operations •C = Geometry

•B = Algebraic Concepts •D = Data Analysis and Probability

These four classifications are used throughout the grade levels. In addition to these classifications, there are five Reporting Categories for each grade level. The first letter of each Reporting Category represents the classification; the second letter represents the Domain as stated in the Common Core State Standards for Mathematics. Listed below are the Reporting Categories for Grade 3.

•A-T = Number and Operations in Base Ten

•A-F = Number and Operations - Fractions

•B-O = Operations and Algebraic Thinking

•C-G = Geometry

•D-M = Measurement and Data

The title of each Reporting Category is consistent with the title of the corresponding Domain in the Common Core State Standards for Mathematics. The Reporting Category title appears at the top of each page.

**Assessment Anchor**

The Assessment Anchor appears in the shaded bar across the top of each Assessment Anchor table. The

Assessment Anchors represent categories of subject matter (skills and concepts) that anchor the content of the PSSA. Each Assessment Anchor is part of a Reporting Category and has one or more Anchor Descriptors unified under and aligned to it.

**Anchor Descriptor**

Below each Assessment Anchor is one or more specific Anchor Descriptors. The Anchor Descriptor adds a level of specificity to the content covered by the Assessment Anchor. Each Anchor Descriptor is part of an Assessment Anchor and has one or more Eligible Content unified under and aligned to it.

**Eligible Content**

The column to the right of the Anchor Descriptor contains the Eligible Content statements. The Eligible

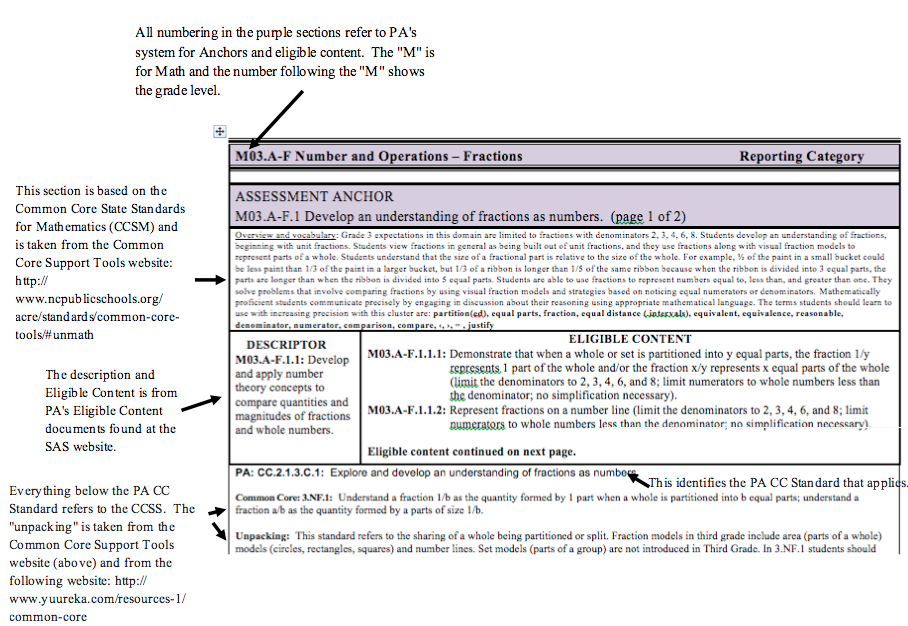
Content is the most specific description of the skills and concepts assessed on the PSSA. This level is considered the assessment limit and helps educators identify the range of the content covered on the PSSA. **Note:** All Grade 3 Eligible Content is considered Non-Calculator.

**Reference**

In the space below each Assessment Anchor table is a code representing one or more Common Core State Standards for Mathematics that correlate to the Eligible Content statements.

Alignment to the “National” Common Core and “unpacking” can be found below the PA CC Standard.

How Do I Read This Document?



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| **M05.A-T Number and Operations in Base Ten Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.A-T.1 Understand the place value system. (Page 1 of 3) | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, decimal, decimal point, patterns, multiply, divide, tenths, thousands, greater than, less than, equal to, ‹, ›, =, compare/comparison, round** | |
| **DESCRIPTOR**  **M05.A-T.1.1:** Demonstrate understanding of place value of whole numbers and decimals, and compare quantities or magnitudes of numbers.  These 4 EC addressed on the next 2 pages. | **ELIGIBLE CONTENT**  **M05.A-T.1.1.1:** Demonstrate an understanding that in a multi-digit number, a digit in one  place represents 1/10 of what it represents in the place to its left. Example:  Recognize that in the number 770, the 7 in the tens place is 1/10 the 7 in the  hundreds place.  **M05.A-T.1.1.2:** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10,  and explain patterns in the placement of the decimal point when a decimal is multiplied or divided  by a power of 10. Use whole-number exponents to denote powers of 10. Example 1: 4 x 102 = 400  Example 2: 0.05 ÷ 103 = 0.00005  **M05.A-T.1.1.3:** Read and write decimals to thousandths using base-ten numerals, word form, and expanded form.  Example: 347.392 = 300 + 40 + 7 + 0.3 + 0.09 + 0.002 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (0.1) + 9 x  (0.01) + 2 x (0.001)  **M05.A-T.1.1.4:** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and  < symbols.  **M05.A-T.1.1.5:** Round decimals to any place (limit rounding to ones, tenths, hundredths, or thousandths place). |
| **PA CC.2.1.5.B.1:** Apply place value concepts to show an understanding of multi-digit whole numbers.  **Common Core: 5.NBT.1:** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.  **Unpacking:** This standard calls for students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is 1/10th the size of the tens place. In fourth grade, students examined the relationships of the digits in numbers for whole numbers only. This standard extends this understanding to the relationship of decimal fractions. Students use base ten blocks, pictures of base ten blocks, and interactive images of base ten blocks to manipulate and investigate the place value relationships. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons. Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and 1/10 of what it represents in the place to its left. Example: The 2 in the number 542 is different from the value of the 2 in 324. The 2 in 542 represents 2 ones or 2, while the 2 in 324 represents 2 tens or 20. Since the 2 in 324 is one place to the left of the 2 in 542 the value of the 2 is 10 times greater. Meanwhile, the 4 in 542 represents 4 tens or 40 and the 4 in 324 represents 4 ones or 4. Since the 4 in 324 is one place to the right of the 4 in 542 the value of the 4 in the number 324 is 1/10th of its value in the number 542. Example: A student thinks, “I know that in the number 5555, the 5 in the tens place (5555) represents 50 and the 5 in the hundreds place (5555) represents 500. So a 5 in the hundreds place is ten times as much as a 5 in the tens place or a 5 in the tens place is 1/10 of the value of a 5 in the hundreds place.  Base on the base-10 number system digits to the left are times as great as digits to the right; likewise, digits to the right are 1/10th of digits to the left. For example, the 8 in 845 has a value of 800 which is ten times as much as the 8 in the number 782. In the same spirit, the 8 in 782 is 1/10th the value of the 8 in 845.  To extend this understanding of place value to their work with decimals, students use a model of one unit; they cut it into 10 equal pieces, shade in, or describe 1/10 of that model using fractional language (“This is 1 out of 10 equal parts. So it is 1/10”. I can write this using 1/10 or 0.1”). They repeat the process by finding 1/10 of a 1/10 (e.g., dividing 1/10 into 10 equal parts to arrive at 1/100 or 0.01) and can explain their reasoning, “0.01 is 1/10 of 1/10 thus is 1/100 of the whole unit.” In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.      The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is 1/10 of 50 and 10 times five tenths.  The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the tenths place is  10 times five hundredths.  THIS ANCHOR CONTINUED ON THE NEXT PAGE | |

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| **M05.A-T Number and Operations in Base Ten Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.A-T.1 Understand the place value system. (Page 2 of 3) | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, decimal, decimal point, patterns, multiply, divide, tenths, thousands, greater than, less than, equal to, ‹, ›, =, compare/comparison, round** | |
| **DESCRIPTOR**  **M05.A-T.1.1:** Demonstrate understanding of place value of whole numbers and decimals, and compare quantities or magnitudes of numbers.  These 3 EC addressed on the next page. | **ELIGIBLE CONTENT**  **M05.A-T.1.1.1:** Demonstrate an understanding that in a multi-digit number, a digit in one place represents 1/10 of  what it represents in the place to its left. Example: Recognize that in the number 770, the 7 in the  tens place is 1/10 the 7 in the hundreds place.  **M05.A-T.1.1.2:** Explain patterns in the number of zeros of the product when multiplying a  number by powers of 10, and explain patterns in the placement of the  decimal point when a decimal is multiplied or divided by a power of 10. Use  whole-number exponents to denote powers of 10. Example 1: 4 x 102 = 400  Example 2: 0.05 ÷ 103 = 0.00005  **M05.A-T.1.1.3:** Read and write decimals to thousandths using base-ten numerals, word form, and expanded form.  Example: 347.392 = 300 + 40 + 7 + 0.3 + 0.09 + 0.002 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (0.1) + 9 x  (0.01) + 2 x (0.001)  **M05.A-T.1.1.4:** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and  < symbols.  **M05.A-T.1.1.5:** Round decimals to any place (limit rounding to ones, tenths, hundredths, or thousandths place). |
| **PA CC.2.1.5.B.1:** Apply place value concepts to show an understanding of multi-digit whole numbers.  **Common Core: 5.NBT.2:** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote  powers of 10.  **Unpacking:** This standard includes multiplying by multiples of 10 and powers of 10, including 102 which is 10 x 10=100, and 103 which is 10 x 10 x 10=1,000. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10. Example: 2.5 x 103 = 2.5 x (10 x 10 x 10) = 2.5 x 1,000 = 2,500 Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right.  350 ÷ 103 = 350 ÷ 1,000 = 0.350 = 0.35 350/10 = 35, 35 /10 = 3.5 3.5 /10 = 0.35, or 350 x 1/10, 35 x 1/10, 3.5 x 1/10 this will relate well to subsequent work with operating with fractions. This example shows that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving (how many times we are dividing by 10 , the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point moves to the left. Students need to be provided with opportunities to explore this concept and come to this understanding; this should not just be taught procedurally.  Example: Students might write:  • 36 x 10 = 36 x 101 = 360  • 36 x 10 x 10 = 36 x 102 = 3600  • 36 x 10 x 10 x 10 = 36 x 103 = 36,000  • 36 x 10 x 10 x 10 x 10 = 36 x 104 = 360,000  Students might think and/or say:  • I noticed that every time, I multiplied by 10 I added a zero to the end of the number. That makes sense because each digit’s value became 10  times larger. To make a digit 10 times larger, I have to move it one place value to the left.  • When I multiplied 36 by 10, the 30 became 300. The 6 became 60 or the 36 became 360. So I had to add a zero at the end to have the 3  represent 3 one-hundreds (instead of 3 tens) and the 6 represents 6 tens (instead of 6 ones). Students should be able to use the same type of  reasoning as above to explain why the following multiplication and division problem by powers of 10 make sense.  • 523 x 103 = 523,000 The place value of 523 is increased by 3 places.  • 5.223 x 102 = 522.3 The place value of 5.223 is increased by 2 places.  • 52.3 ÷ 101 = 5.23 The place value of 52.3 is decreased by one place.    •Multiply a number by 10, by 100, by 1000, etc., and observe and explain patterns in the number of zero in the product.  •Multiply a decimal by 10, by 100, by 1000, etc., and observe and explain patterns in the placement of the decimal point.  •Divide a number or decimal by 10, by 100, by 1000, etc., and observe and explain patterns in the placement of the decimal point.  •Use 101 to denote multiplying by 10, 102 to denote multiplying by 100, 103 to denote multiplying by 1000, etc., and explain patterns in the  placement of the decimal point in relation to the whole-number exponent of 10.  THIS ANCHOR CONTINUED ON THE NEXT PAGE | |

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| **M05.A-T Number and Operations in Base Ten Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.A-T.1 Understand the place value system. (Page 3 of 3) | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, decimal, decimal point, patterns, multiply, divide, tenths, thousands, greater than, less than, equal to, ‹, ›, =, compare/comparison, round** | |
| **DESCRIPTOR**  **M05.A-T.1.1:** Demonstrate understanding of place value of whole numbers and decimals, and compare quantities or magnitudes of numbers. | **ELIGIBLE CONTENT**  **M05.A-T.1.1.1:** Demonstrate an understanding that in a multi-digit number, a digit in one place represents 1/10 of  what it represents in the place to its left. Example: Recognize that in the number 770, the 7 in the  tens place is 1/10 the 7 in the hundreds place.  **M05.A-T.1.1.2:** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10,  and explain patterns in the placement of the decimal point when a decimal is multiplied or divided  by a power of 10. Use whole-number exponents to denote powers of 10. Example 1: 4 x 102 = 400  Example 2: 0.05 ÷ 103 = 0.00005  **M05.A-T.1.1.3:** Read and write decimals to thousandths using base-ten numerals, word form,  and expanded form. Example: 347.392 = 300 + 40 + 7 + 0.3 + 0.09 + 0.002  = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (0.1) + 9 x (0.01) + 2 x (0.001)  **M05.A-T.1.1.4:** Compare two decimals to thousandths based on meanings of the digits in  each place, using >, =, and < symbols.  **M05.A-T.1.1.5:** Round decimals to any place (limit rounding to ones, tenths, hundredths, or  thousandths place). |
| **PA CC.2.1.5.B.1:** Apply place value concepts to show an understanding of multi-digit whole numbers.  **Common Core: 5.NBT.3:** Read, write, and compare decimals to thousandths.  a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10  + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000).  b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of  comparisons.  **Unpacking:** •Read and write decimals to the thousandths and go back and forth between each of these forms:  -Base-10 numbers (e.g., 14.56) -Number names (e.g., Fourteen and fifty-six hundredths)  -Expanded form (e.g., 14.56 = 1 . 10 + 4 . 1 + 5 . (1/10) + 6 . (1/100)  •Compare two decimals and use >, =, and < symbols to record the results of comparisons.  This standard references expanded form of decimals with fractions included. Students should build on their work from Fourth Grade, where they worked with both decimals and fractions interchangeably. Expanded form is included to build upon work in 5.NBT.2 and deepen students’ understanding of place value. Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation. This investigation leads them to understanding equivalence of decimals (0.8 = 0.80 = 0.800). Comparing decimals builds on work from fourth grade. Example: Some equivalent forms of 0.72 are: 72/100, 7/10 + 2/100, 0.70 + 0.02,  7 x (1/10) + 2 x (1/100), 70/100 + 2/100, 0.720, 7 x (1/10) + 2 x (1/100) + 0 x (1/1000), 720/1000 Students need to understand the size of decimal numbers and relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals. Example: Comparing 0.25 and 0.17, a student might think, “25 hundredths is more than 17 hundredths”. They may also think that it is 8 hundredths more. They may write this comparison as 0.25 > 0.17 and recognize that 0.17 < 0.25 is another way to express this comparison. Comparing 0.207 to 0.26, a student might think, “Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths so the second number must be larger. Another student might think while writing fractions, “I know that 0.207 is 207 thousandths (and may write 207/1000). 0.26 is 26 hundredths (and may write 26/100) but I can also think of it as 260 thousandths (260/1000). So, 260 thousandths is more than 207 thousandths.  **Common Core: 5.NBT.4:** Use place value understanding to round decimals to any place.  **Unpacking:** Round any decimal to a given place by looking at the place to the right to decide whether to round up or round down. This standard refers to rounding. Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding. Example: Round 14.235 to the nearest tenth. Students recognize that the possible answer must be in tenths thus, it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).  Students should use benchmark numbers to support this work. Benchmarks are convenient numbers for  comparing and rounding numbers. 0., 0.5, 1, 1.5 are examples of benchmark numbers. | |

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| **M05.A-T Number and Operations in Base Ten Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.A-T.2 Perform operations with multi-digit whole numbers and with decimals to hundredths.  (Page 1 of 3) | |
| Overview and vocabulary: Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/division, decimal, decimal point, tenths, hundredths, products, quotients, dividends, rectangular arrays, area models, addition/add, subtraction/subtract, (properties)-rules about how numbers work, reasoning** | |
| **DESCRIPTOR**  M05.A-T.2.1: Use whole numbers and decimals to compute accurately (straight computation or word problems). | **ELIGIBLE CONTENT**  **M05.A-T.2.1.1:** Multiply multi-digit whole numbers (not to exceed 3-digit by 3-digit).  **M05.A-T.2.1.2:** Find whole-number quotients of whole numbers with up to four-digit  dividends and two-digit divisors.  **M05.A-T.2.1.3:** Add, subtract, multiply, and divide decimals to hundredths (no divisors with  decimals). |
| **PA: CC.2.1.5.B.2:**  Extend an understanding of operations with whole numbers to perform operations including decimals.  **Common Core: 5.NBT.5:** Fluently multiply multi-digit whole numbers using the standard algorithm.  **Unpacking:** This standard refers to fluency which means accuracy (correct answer), efficiency (a reasonable amount of steps), and flexibility (using strategies such as the distributive property or breaking numbers apart also using strategies according to the numbers in the problem, 26 x 4 may lend itself to (25 x 4 ) + 4 where as another problem might lend itself to making an equivalent problem 32 x 4 = 64 x 2)). This standard builds upon students’ work with multiplying numbers in third and fourth grade. In fourth grade, students developed understanding of multiplication through using various strategies. While the standard algorithm is mentioned, alternative strategies are also appropriate to help students develop conceptual understanding. The size of the numbers should NOT exceed a three-digit factor by a two-digit factor. Examples of alternative strategies:  There are 225 dozen cookies in the bakery. How many cookies are there?    Draw a array model for 225 x 12…. 200 x 10, 200 x 2, 20 x 10, 20 x 2, 5 x 10, 5 x 2    THIS ANCHOR IS CONTINED ON THE NEXT PAGE | |

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| **M05.A-T Number and Operations in Base Ten Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.A-T.2 Perform operations with multi-digit whole numbers and with decimals to hundredths.  (Page 2 of 3) | |
| **DESCRIPTOR**  M05.A-T.2.1: Use whole numbers and decimals to compute accurately (straight computation or word problems). | **ELIGIBLE CONTENT**  **M05.A-T.2.1.1:** Multiply multi-digit whole numbers (not to exceed 3-digit by 3-digit).  **M05.A-T.2.1.2:** Find whole-number quotients of whole numbers with up to four-digit  dividends and two-digit divisors.  **M05.A-T.2.1.3:** Add, subtract, multiply, and divide decimals to hundredths (no divisors with  decimals). |
| **PA: CC.2.1.5.B.2:**  Extend an understanding of operations with whole numbers to perform operations including decimals.  **Common Core: 5.NBT.6:** Find whole-number quotients of whole numbers with up to four-digit dividendsand two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.  **Unpacking:** This standard references various strategies for division. Division problems can include remainders. Even though this standard leads more towards computation, the connection to story contexts is critical. Make sure students are exposed to problems where the divisor is the number of groups and where the divisor is the size of the groups. In fourth grade, students’ experiences with division were limited to dividing by one-digit divisors. This standard extends students’ prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a “familiar” number, a student might decompose the dividend using place value. Example: There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams get created? If you have left over students, what do you do with them?  **Student 1**  1,716 divided by 16 There are 100 16’s in 1,716. 1,716 – 1,600 = 116 I know there are at least 6 16’s. 116 - 96 = 20  I can take out at least 1 more 16.  20 - 16 = 4 There were 107 teams with 4 students left over. If we put the extra students on different team, 4 teams will have 17 students.      Example:Using expanded notation 2682 ÷ 25 = (2000 + 600 + 80 + 2) ÷ 25      Using understanding of the relationship between 100 and 25, a student might think ~ • I know that 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80. • 600 divided by 25 has to be 24. • Since 3 x 25 is 75, I know that 80 divided by 25 is 3 with a reminder of 5.  (Note that a student might divide into 82 and not 80) • I can’t divide 2 by 25 so 2 plus the 5 leaves a remainder of 7. • 80 + 24 + 3 = 107. So, the answer is 107 with a remainder of 7. Using an equation that relates division to multiplication, 25 x n = 2682, a student might estimate the answer to be slightly larger than 100 because s/he recognizes that 25 x 100 = 2500.    Example: 968 ÷ 21 Using base ten models, a student can represent 962 and use the models to make an array with one dimension of 21. The student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array.    Example: 9984 ÷ 64  An area model for division is shown below. As the student uses the area model, s/he keeps track of how much of the 9984 is left to divide.  THIS ANCHOR IS CONTINED ON THE NEXT PAGE | |

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| **M05.A-T Number and Operations in Base Ten Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.A-T.2 Perform operations with multi-digit whole numbers and with decimals to hundredths.  (Page 3 of 3) | |
| **DESCRIPTOR**  M05.A-T.2.1: Use whole numbers and decimals to compute accurately (straight computation or word problems). | **ELIGIBLE CONTENT**  **M05.A-T.2.1.1:** Multiply multi-digit whole numbers (not to exceed 3-digit by 3-digit).  **M05.A-T.2.1.2:** Find whole-number quotients of whole numbers with up to four-digit  dividends and two-digit divisors.  **M05.A-T.2.1.3:** Add, subtract, multiply, and divide decimals to hundredths (no divisors with  decimals). |
| **PA: CC.2.1.5.B.2:**  Extend an understanding of operations with whole numbers to perform operations including decimals.  **Common Core: 5.NBT.7:** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.  **Unpacking:** •Add, subtract, multiply, and divide decimals to hundredths. •Use drawings to represent the strategy used. •Use numbers/symbols and/or a written explanation to explain the reasoning behind the strategy that is used. •Use strategies based on place value, properties of operations, and the relationship between addition and subtraction such as by using:  - Concrete base-10 model (e.g., a small square on a 10.10 grid could represent one-hundredths, and one row of the 10.10 grid could  represent one-tenths);  - Rectangular arrays or area models to illustrate multiplication or division, or  - Equations or expressions to represent a series of steps that leads to the answer.  **NOTE**: The standard algorithm is NOT a focus of this Standard, but rather, an understanding of what it means to perform operations with decimals. Proficiency with the standard algorithms with decimals will be in 6th grade. This standard builds on the work from fourth grade where students are introduced to decimals and compare them. In fifth grade, students begin adding, subtracting, multiplying and dividing decimals. This work should focus on concrete models and pictorial representations, rather than relying solely on the algorithm. The use of symbolic notations involves having students record the answers to computations (2.25 x 3= 6.75), but this work should not be done without models or pictures. This standard includes students’ reasoning and explanations of how they use models, pictures, and strategies. This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.  Examples: • 3.6 + 1.7 A student might estimate the sum to be larger than 5 because 3.6 is more than 3 ½ and 1.7 is more than 1 ½ .  • 5.4 – 0.8 A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.  • 6 x 2.4 A student might estimate an answer between 12 and 18 since 6 x 2 is 12 and 6 x 3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than 6 x 2 ½ and think of 2 ½ groups of 6 as 12 (2 groups of 6) + 3 ( ½ of a group of 6). Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.    Example: A recipe for a cake requires 1.25 cups of milk, 0.40 cups of oil, and 0.75 cups of water. How much liquid is in the mixing bowl?  **Student 1**  1.25 + 0.40 + 0.75  First, I broke the numbers apart:  I broke 1.25 into 1.00 + 0.20 + 0.05  I left 0.40 like it was.  I broke 0.75 into 0.70 + 0.05  I combined my two 0.05s to get 0.10.  I combined 0.40 and 0.20 to get 0.60.  I added the 1 whole from 1.25.  I ended up with 1 whole, 6  tenths, 7 more tenths and 1  more tenth which equals 2.40  **Student 2**  I saw that the 0.25 in 1.25 and the 0.75 for water would combine to equal 1 whole. I then added the 2 wholes and the 0.40 to get 2.40. | |

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| **M05.A-F Number and Operations – Fractions**  **Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.A-F.1 Use equivalent fractions as a strategy to add and subtract fractions. | |
| Overview and vocabulary: Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, equivalent, addition/ add, sum, subtraction/subtract, difference, unlike denominator, numerator, benchmark fraction, estimate, reasonableness, mixed numbers** | |
| **DESCRIPTOR**  **M05.A-F.1.1:** Solve addition and subtraction problems involving fractions (straight computation or word problems). | **ELIGIBLE CONTENT**  **M05.A-F.1.1.1:** Add and subtract fractions (including mixed numbers) with unlike  denominators. (May include multiple methods and  representations.) Example: 2/3 + 5/4 = 8/12 + 15/12 = 23/12 |
| **PA: CC.2.1.5.C.1:**  Use the understanding of equivalency to add and subtract fractions.  **Common Core: 5.NF.1**: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.  *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad +bc)/bd.)*  **Unpacking:** This builds on the work in fourth grade where students add fractions with like denominators. In fifth grade, the example provided in the standard has students find a common denominator by finding the product of both denominators. For 1/3 + 1/6, a common denominator is 18, which is the product of 3 and 6. This process should be introduced using visual fraction models (area models, number lines, etc.) to build understanding before moving into the standard algorithm. Students should apply their understanding of equivalent fractions and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the smallest denominator.    **Common Core: 5.NF.2:** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.    Example:  Present students with the problem 1/3 + 1/6. Encourage students to use the clock face as a model for solving the problem. Have students share their approaches with the class and demonstrate their thinking using the clock  model.  **Unpacking:** This standard refers to number sense, which means students’ understanding of fractions as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents, also being able to use reasoning such as 7/8 is greater than ¾ because 7/8 is missing only 1/8 and ¾ is missing ¼ so 7/8 is closer to a whole Also, students should use benchmark fractions to estimate and examine the reasonableness of their answers. Example here such as 5/8 is greater than 6/10 because 5/8 is 1/8  larger than ½ (4/8) and 6/10 is only 1/10 larger than ½ (5/10)  When presented with word problems involving addition and subtraction of fractions referring to the same whole:  •Represent the problem with an equation using symbols (such as a blank or empty box or question mark) to represent the unknown values.  •Use visual fraction models or drawings that appropriately models the situation described in the problem.  •Find the correct solution by adding/subtracting the fractions with like or unlike denominators (see 5.NF.1).  •Interpret the solution in the context of the problem, especially explaining what the remainder means in a division problem.  •Reflect on the reasonableness of the answer by using estimation strategies (such as estimating to the nearest whole number or benchmark fraction).  Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies for calculations with fractions extend from students’ work with whole number operations and can be supported through the use of physical models.  Example: Elli drank 3/5 quart of milk and Javier drank 1/10 of a quart less than Ellie. How much milk did they drink all together?  Together they drank 1 1/10 quarts of milk.  This is how much milk Javier drank    This solution is reasonable because Ellie drank more than ½ quart and Javier drank ½ quart so together they drank slightly more than one qt. | |

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| **M05.A-F Number and Operations – Fractions**  **Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.A-F.2 Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Page 1 of 4) | |
| Overview and vocabulary: Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.) Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional sides lengths, scaling, comparing** | |
| **DESCRIPTOR**  **M05.A-F.2.1:** Solve multiplication and division problems involving fractions and whole numbers (straight computation or word problems).  These 3 EC addressed on the next 3 pages | **ELIGIBLE CONTENT**  **M05.A-F.2.1.1:** Solve word problems involving division of whole numbers leading  to answers in the form of fractions (including mixed numbers).  **M05.A-F.2.1.2:** Multiply a fraction (including mixed numbers) by a fraction.  **M05.A-F.2.1.3:** Demonstrate an understanding of multiplication as scaling (resizing). Example 1:  Comparing the size of a product to the size of one factor on the basis of the size of the  other factor, without performing the indicated multiplication. Example 2: Explaining  why multiplying a given number by a fraction greater than 1 results in a product  greater than the given number (recognizing multiplication by whole numbers greater  than 1 as a familiar case); explaining why multiplying a given number by a fraction  less than 1 results in a product smaller than the given number.  **M05.A-F.2.1.4:** Divide unit fractions by whole numbers and whole numbers by unit fractions. |
| **PA: CC.2.1.5.C.2:** Apply and extend previous understandings of multiplication and division to multiply and divide fractions.  **Common Core: 5.NF.3**: Interpret a fraction as division of the numerator by the denominator (*a*/*b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*  **Unpacking:**  •Explain the relationship between and by drawing a picture or describing a context.  When presented with word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers:  • Represent the problem with an equation using symbols (such as a blank or empty box or question mark) to represent the unknown values.  • Use visual fraction models or drawings that appropriately model the situation described in the problem.  • Find the correct solution using a division strategy.  • Interpret the solution in the context of the problem, especially explaining what fractional part of the answer represents.  This standard calls for students to extend their work of partitioning a number line from third and fourth grade. Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities. Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read 3/5 as “three fifths” and after many experiences with sharing problems, learn that 3/5 can also be interpreted as “3 divided by 5.” Examples: Ten team members are sharing 3 boxes of cookies. How much of a box will each student get? When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so s/he is seeing the solution to the following equation, 10 x n = 3 (10 groups of some amount is 3 boxes) which can also be written as n = 3 ÷ 10. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting 3/10 of a box. Two afterschool clubs are having pizza parties. For the Math Club, the teacher will order 3 pizzas for every 5 students. For the student council, the teacher will order 5 pizzas for every 8 students. Since you are in both groups, you need to decide which party to attend. How much pizza would you get at each party? If you want to have the most pizza, which party should you attend? The six fifth grade classrooms have a total of 27 boxes of pencils. How many boxes will each classroom receive? Students may recognize this as a whole number division problem but should also express this equal sharing problem as 27/6. They explain that each classroom gets 27/6 boxes of pencils and can further determine that each classroom get 4 3/6 or 4 ½ boxes of pencils.  Example:  Your teacher gives 7 packs of paper to your group of 4 students. If you share the paper equally, how much paper does each student get?    Each student receives 1 whole pack of paper and . of the each of the 3 packs of paper. So each student gets 1 . packs of paper.    THIS ANCHOR CONTINUED ON NEXT PAGE | |

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| **M05.A-F Number and Operations – Fractions**  **Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.A-F.2 Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Page 2 of 4) | |
| **DESCRIPTOR**  **M05.A-F.2.1:** Solve multiplication and division problems involving fractions and whole numbers (straight computation or word problems).  These 2 EC addressed on the next 2 pages. | **ELIGIBLE CONTENT**  **M05.A-F.2.1.1:** Solve word problems involving division of whole numbers leading to answers in the form  of fractions (including mixed numbers).  **M05.A-F.2.1.2:** Multiply a fraction (including mixed numbers) by a fraction.  **M05.A-F.2.1.3:** Demonstrate an understanding of multiplication as scaling (resizing). Example 1: Comparing the size of  a product to the size of one factor on the basis of the size of the other factor, without performing the  indicated multiplication. Example 2: Explaining why multiplying a given number by a fraction greater  than 1 results in a product greater than the given number (recognizing multiplication by whole numbers  greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1  results in a product smaller than the given number.  **M05.A-F.2.1.4:** Divide unit fractions by whole numbers and whole numbers by unit fractions. |
| **PA:CC.2.1.5.C.2:**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.  **Common Core: 5.NF.4**: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.  a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b.  For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) =  8/15. (In general, (a/b) × (c/d) = ac/bd.)  b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that  the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and r  represent fraction products as rectangular areas.  **Unpacking:**  Students need to develop a fundamental understanding that the multiplication of a fraction by a whole number could be represented as repeated addition of a unit fraction (e.g., 2 x (1/4) = 1/4 + ¼ This standard extends student’s work of multiplication from earlier grades. In fourth grade, students worked with recognizing that a fraction such as 3/5 actually could be represented as 3 pieces that are each one-fifth (3 x (1/5)).  This standard references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard. As they multiply fractions such as 3/5 x 6, they can think of the operation in more than one way. • 3 x (6 ÷ 5) or (3 x 6/5) • (3 x 6) ÷ 5 or 18 ÷ 5 (18/5) Students create a story problem for 3/5 x 6 such as, • Isabel had 6 feet of wrapping paper. She used 3/5 of the paper to wrap some presents. How much does she have left? • Every day Tim ran 3/5 of mile. How far did he run after 6 days? (Interpreting this as 6 x 3/5)  Example: Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?  This question is asking what 2/3 of ¾ is, or what is 2/3 x ¾ . What is 2/3 x ¾, in this case you have 2/3 groups of size ¾ ( a way to think about it in terms of the language for whole numbers is 4 x 5 you have 4 groups of size 5. The array model is very transferable from whole number work and then to binomials.  **Student 1**  I drew a rectangle to represent the whole class. The four columns represent the fourths of a class. I shaded 3 columns to represent the fraction that are boys. I then split the rectangle with horizontal lines into thirds. The dark area represents the fraction of the boys in the class wearing tennis shoes, which is 6 out of 12. That is 6/12, which equals 1/2.          This standard extends students’ work with area. In third grade students determine the area of rectangles and composite rectangles. In fourth grade students continue this work. The fifth grade standard calls students to continue the process of covering (with tiles). Grids (see picture) below can be used to support this work.  Example: The home builder needs to cover a small storage room floor with carpet. The storage room is 4 meters long and half of a meter wide. How much carpet do you need to cover the floor of the storage room? Use a grid to show your work and explain your answer. In the grid below I shaded the top half of 4 boxes. When I added them together, I added ½ four times, which equals 2. I could also think about this with multiplication ½ x 4 is equal to 4/2 which is equal to 2.  2/3 x 4/5  The area model and the line segments show that the area is the same quantity as the product of the side lengths.  THIS ANCHOR CONTINUED ON NEXT PAGE | |

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| **M05.A-F Number and Operations – Fractions**  **Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.A-F.2 Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Page 3 of 4) | |
| **DESCRIPTOR**  **M05.A-F.2.1:** Solve multiplication and division problems involving fractions and whole numbers (straight computation or word problems).  This EC addressed on the  next page. | **ELIGIBLE CONTENT**  **M05.A-F.2.1.1:** Solve word problems involving division of whole numbers leading to answers in the form  of fractions (including mixed numbers).  **M05.A-F.2.1.2:** Multiply a fraction (including mixed numbers) by a fraction.  **M05.A-F.2.1.3:** Demonstrate an understanding of multiplication as scaling (resizing). Example  1: Comparing the size of a product to the size of one factor on the basis of the  size of the other factor, without performing the indicated multiplication.  Example 2: Explaining why multiplying a given number by a fraction greater  than 1 results in a product greater than the given number (recognizing  multiplication by whole numbers greater than 1 as a familiar case); explaining  why multiplying a given number by a fraction less than 1results in a product  smaller than the given number.  **M05.A-F.2.1.4:** Divide unit fractions by whole numbers and whole numbers by unit fractions. |
| **PA:CC.2.1.5.C.2:**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.  **Common Core: 5.NF.5**: Interpret multiplication as scaling (resizing), by:  a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated  multiplication.  b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing  multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1  results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of  multiplying a/b by 1.  **Unpacking: •**When multiplying two numbers (including fractions and decimals):  -Say whether the size of the product will be greater than or less than the size of one of the factors.  -Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (e.g., when  multiplying 4 x 2 1/8 the student knows that the product will be more than 4 without performing a calculation);  -Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number (e.g., when multiplying  -Explain why multiplying the same number to the numerator and denominator    results in an equivalent fraction    4 x 1/8 the student knows that the product will be less than 4 without performing a calculation);    This standard calls for students to examine the magnitude of products in terms of the relationship between two types of problems. This extends the work with 5.OA.1.    This standard asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less the one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4, and should not be taught in isolation.  Example: Mrs. Bennett is planting two flower beds. The first flower bed is 5 meters long and 6/5 meters wide. The second flower bed is 5 meters long and 5/6 meters wide. How do the areas of these two flower beds compare? Is the value of the area larger or smaller than 5 square meters? Draw pictures to prove your answer.  THIS ANCHOR CONTINUED ON NEXT PAGE | |

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| **M05.A-F Number and Operations – Fractions**  **Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.A-F.2 Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Page 4 of 4) | |
| **DESCRIPTOR**  **M05.A-F.2.1:** Solve multiplication and division problems involving fractions and whole numbers (straight computation or word problems). | **ELIGIBLE CONTENT**  **M05.A-F.2.1.1:** Solve word problems involving division of whole numbers leading to answers in the form  of fractions (including mixed numbers).  **M05.A-F.2.1.2:** Multiply a fraction (including mixed numbers) by a fraction.  **M05.A-F.2.1.3:** Demonstrate an understanding of multiplication as scaling (resizing). Example 1: Comparing the size of  a product to the size of one factor on the basis of the size of the other factor, without performing the  indicated multiplication. Example 2: Explaining why multiplying a given number by a fraction greater  than 1 results in a product greater than the given number (recognizing multiplication by whole numbers  greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1  results in a product smaller than the given number.  **M05.A-F.2.1.4:** Divide unit fractions by whole numbers and whole numbers by unit fractions. |
| **PA:CC.2.1.5.C.2:**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.  **Common Core: 5.NF.7**: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Note: Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.)  a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4,  and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12  because (1/12) x 4 = 1/3.  b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a  visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because  20 x (1/5) = 4.  c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions,  e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people  share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?  **Unpacking:** This standard is the first time that students are dividing with fractions. In fourth grade students divided whole numbers, and multiplied a whole number by a fraction. The concept unit fraction is a fraction that has a one in the denominator. For example, the fraction 3/5 is 3 copies of the unit fraction 1/5. 1/5 + 1/5 + 1/5 = 3/5 = 1/5 x 3 or 3 x 1/5 Example: Knowing the number of groups/shares and finding how many/much in each group/share.Four students sitting at a table were given 1/3 of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally? The diagram shows the 1/3 pan divided into 4 equal shares with each share equaling 1/12 of the pan.    This standard asks students to work with story contexts where a unit fraction is divided by a non-zero whole number. Students should use various  fraction models and reasoning about fractions. Example: You have 1/8 of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?    •For division of a unit fraction (1/b) by a non-zero whole number:  - Create a story context for the expression; and - Compute the quotient.  •For division of a whole number by a unit fraction (1/b):  - Create a story context for the expression; and - Compute the quotient.  •When presented with word problems involving division of a unit fraction by a non-zero whole number (1/b ÷ n), or division of a whole number by a unit fraction (n ÷ 1/b):  - Represent the problem with an equation using symbols (such as a blank or empty box or question mark) to represent the unknown values.  - Use visual fraction models or drawings that appropriately model the situation described in the problem.  - Find the correct solution using a division strategy.  - Interpret the solution in the context of the problem, especially explaining what fractional part of the answer represents. | |

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| **M05.A-F Number and Operations – Fractions**  **Reporting Category** |
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| ASSESSMENT ANCHOR  PA does not have an anchor that correlates specifically to this “National” Common Core Standard. |
| **Common Core: 5.NF.6**: Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.  Unpacking:  When presented with word problems involving multiplication of fractions and mixed numbers, the student can:  •Represent the problem with an equation using symbols (such as a blank or empty box or question mark) to represent the unknown values.  •Use visual fraction models or drawings that appropriately model the situation described in the problem.  •Find the correct solution using a multiplication strategy.  •Interpret the solution in the context of the problem, especially explaining what fractional part of the answer represents.  This standard builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number. Example: There are 2 ½ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. 2/5 of the students on each bus are girls. How many busses would it take to carry only the girls?    Example:  Evan bought 6 roses for his mother. 2/3 of them were red. How many red roses were there? Using a visual, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups. |

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| **M05.B-O Operations and Algebraic Thinking Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.B-O.1 Write and interpret numerical expressions. | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **parentheses, brackets, braces, numerical expressions** | |
| **DESCRIPTOR**  **M05.B-O.1.1** Analyze and complete  calculations by applying the  order of operations. | **ELIGIBLE CONTENT**  **M05.B-O.1.1.1:** Use multiple grouping symbols (parentheses, brackets, or braces) in numerical expressions, a  and evaluate expressions containing these symbols.  **M05.B-O.1.1.2:** Write simple expressions that model calculations with numbers, and interpret numerical  expressions without evaluating them. Example 1: Express the calculation “add 8 and 7, then  multiply by 2” as 2 x (8 + 7). Example 2: Recognize that 3 x (18,932 + 921) is three times as  large as 18,932 + 921, without having to calculate the indicated sum or product. |
| **PA:CC.2.2.5.A.1:** Interpret and evaluate numerical expressions using order of operations.  **Common Core: 5.OA.1:** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.  **Unpacking:** The standard calls for students to evaluate expressions with parentheses ( ), brackets [ ] and braces { }. In upper levels of mathematics, evaluate means to substitute for a variable and simplify the expression. However at this level students are to only simplify the expressions because there are no variables. Example: Evaluate the expression 2{ 5[12 + 5(500 - 100) + 399]} Students should have experiences working with the order of first evaluating terms in parentheses, then brackets, and then braces. The first step would be to subtract 500 – 100 = 400. Then multiply 400 by 5 = 2,000. Inside the bracket, there is now [12 + 2,000 + 399]. That equals 2,411. Next multiply by the 5 outside of the bracket. 2,411 x 5 = 12,055. Next multiply by the 2 outside of the braces. 12,055 x 2= 24,110. Mathematically, there cannot be brackets or braces in a problem that does not have parentheses. Likewise, there cannot be braces in a problem that does not have both parentheses and brackets. This standard builds on the expectations of third grade where students are expected to start learning the conventional order. Students need experiences with multiple expressions that use grouping symbols throughout the year to develop understanding of when and how to use parentheses, brackets, and braces. First, students use these symbols with whole numbers. Then the symbols can be used as students add, subtract, multiply and divide decimals and fractions.  Example:  • (26 + 18) 4 Solution: 11 • {[2 x (3+5)] – 9} + [5 x (23-18)] Solution: 32 • 12 – (0.4 x 2) Solution: 11.2 • (2 + 3) x (1.5 – 0.5) Solution: 5  To further develop students’ understanding of grouping symbols and facility with operations, students place grouping symbols in equations to make the equations true or they compare expressions that are grouped differently.  Example:  • 15 - 7 – 2 = 10 → 15 - (7 – 2) = 10  • 3 x 125 ÷ 25 + 7 = 22 → [3 x (125 ÷ 25)] + 7 = 22  • 24 ÷ 12 ÷ 6 ÷ 2 = 2 x 9 + 3 ÷ ½ → 24 ÷ [(12 ÷ 6) ÷ 2] = (2 x 9) + (3 ÷ ½ )  • Compare 3 x 2 + 5 and 3 x (2 + 5)  • Compare 15 – 6 + 7 and 15 – (6 + 7)  **Common Core: 5.OA.2**:Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.  **Unpacking:** This standard refers to expressions. Expressions are a series of numbers and symbols (+, -, x, ÷) without an equal sign. Equations result when two expressions are set equal to each other (2 + 3 = 4 + 1). Example: 4(5 + 3) is an expression. When we compute 4(5 + 3) we are evaluating the expression. The expression equals 32. 4(5 + 3) = 32 is an equation. This standard calls for students to verbally describe the relationship between expressions without actually calculating them. This standard calls for students to apply their reasoning of the four operations as well as place value while describing the relationship between numbers. The standard does not include the use of variables, only numbers and signs for operations.  Example: Write an expression for the steps “double five and then add 26.” Solution: (2 x 5) + 26  Example: Describe how the expression 5(10 x 10) relates to 10 x 10. Solution: The expression 5(10 x 10) is 5 times larger than the expression 10 x 10 since I know that I that 5(10 x 10) means that I have 5 groups of (10 x 10).  •Translate a verbal description of a calculation into a numerical expression.  •Interpret a numerical expression and describe it without performing a calculation. | |

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| **M05.B-O Operations and Algebraic Thinking Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.B-O.2 Analyze patterns and relationships. | |
| Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **numerical patterns, rules, ordered pairs, coordinate plane** | |
| **DESCRIPTOR**  **M05.B-O.2.1:** Create, extend, and analyze  patterns. | **ELIGIBLE CONTENT**  **M05.B-O.2.1.1:** Generate two numerical patterns using two given rules. Example:  Given the rule “Add 3” and the starting number 0, and given the  rule “Add 6” and the starting number 0, generate terms in the  resulting sequences.  **M05.B-O.2.1.2:** Identify apparent relationships between corresponding terms of two  patterns with the same starting numbers that follow different rules.  Example: Given two patterns in which the first pattern follows the  rule “add 8” and the second pattern follows the rule “add 2,”  observe that the terms in the first pattern are 4 times the size of the  terms in the second pattern. |
| **PA:CC.2.2.5.A.4:** Analyze patterns and relationships using two rules.  **Common Core: 5.OA.3:** Generatetwo numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*  **Unpacking:**  When given two rules:   Generate a numerical pattern for each rule;   Form ordered pairs by taking pairing the 1st term from each pattern, then the 2nd term from each pattern, etc.;   Graph the ordered pairs on a coordinate plane;   Identify any relationship or pattern in the ordered pairs or in the graph.  This standard extends the work from Fourth Grade, where students generate numerical patterns when they are given one rule. In Fifth Grade, students are given two rules and generate two numerical patterns. The graphs that are created should be line graphs to represent the pattern. This is a linear function which is why we get the straight lines. The Days are the independent variable, Fish are the dependent variables, and the constant rate is what the rule identifies in the table. Example: Make a chart (table) to represent the number of fish that Sam and Terri catch.     |  |  |  | | --- | --- | --- | | Days | Sam’s Total Number of Fish | Terri’s Total Number of Fish | | 0 | 0 | 0 | | 1 | 2 | 4 | | 2 | 4 | 8 | | 3 | 6 | 12 | | 4 | 8 | 16 | | 5 | 10 | 20 |   Describe the pattern: Since Terri catches 4 fish each day, and Sam catches 2 fish, the amount of Terri’s fish is always greater. Terri’s fish is also always twice as much as Sam’s fish. Today, both Sam and Terri have no fish. They both go fishing each day. Sam catches 2 fish each day. Terri catches 4 fish each day. How many fish do they have after each of the five days? Make a graph of the number of fish. | |

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| **M05.C-G Geometry Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.C-G.1 Graph points on the coordinate plane to solve real-world and mathematical problems. | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **coordinate system, coordinate plane, first quadrant, points, lines, axis/axes, x-axis, y-axis, horizontal, vertical, intersection of lines, origin, ordered pairs, coordinates, x-coordinate, y-coordinate** | |
| **DESCRIPTOR**  **M05.C-G.1.1:** Identify parts of a coordinate grid, and describe or interpret points given an ordered pair. | **ELIGIBLE CONTENT**  **M05.C-G.1.1.1:** Identify parts of the coordinate plane (**x**-axis, **y**-axis, and the origin) and the  ordered pair (**x**-coordinate and **y**-coordinate). Limit the coordinate plane to  quadrant I.  **M05.C-G.1.1.2**: Represent real-world and mathematical problems by plotting points in quadrant I  of the coordinate plane, and interpret coordinate values of points in the context of  the situation. |
| **PA:CC.2.3.5.A.1:** Graph points in the first quadrant on the coordinate plane and interpret these points when solving real world and mathematical problems.  **Common Core: 5.G.1:** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., **x**-axis and **x**-coordinate, **y**-axis and **y**-coordinate).  **Unpacking:** These standards deal with only the first quadrant (positive numbers) in the coordinate plane.    • Construct a coordinate system, identify the origin, appropriately scale each axis, and label each axis.  • Recognize that the horizontal axis is generally labeled as the *x*-axis, and the vertical axis is generally labeled as the *y*-axis.  • Recognize that ordered pairs are generally in the form of (*x*, y) in which the first number (*x-*coordinate) indicates how far to travel from the origin  along the x-axis, and the second number (*y*-coordinate) indicates how far to travel in the direction of the *y*-axis.  **Common Core: 5.G.2**: Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.  **Unpacking:** 5.G.2 references real-world and mathematical problems, including the traveling from one point to another and identifying the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms. When given a real world or mathematical problem that involves graphing points in the first quadrant:  • Generate or identify ordered pairs.  • Construct a coordinate system that accommodates the ordered pairs.  • Graph the ordered pairs.  • Interpret the coordinate values of points in the context of the problem. | |

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| **M05.C-G Geometry Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.C-G.2 Classify two-dimensional figures into categories based on their properties. | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: attribute, category, subcategory, hierarchy, (properties)-rules about how numbers work, two dimensional From previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle** | |
| **DESCRIPTOR**  **M05.C-G.2.1:** Use basic properties to classify  two-dimensional figures. | **ELIGIBLE CONTENT**  **M05.C-G.2.1.1:** Classify two-dimensional figures in a hierarchy based on properties. Example 1:  All polygons have at least 3 sides, and pentagons are polygons, so all pentagons  have at least 3 sides. Example 2: A rectangle is a parallelogram, which is a  quadrilateral, which is a polygon; so, a rectangle can be classified as a  parallelogram, as a quadrilateral, and as a polygon. |
| **PA:CC.2.3.5.A.2:** Classify two-dimensional figures into categories based on an understanding of their properties.  **Common Core: 5.G.3:** Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.  **Unpacking:** This standard calls for students to reason about the attributes (properties) of shapes. Student should have experiences discussing the property of shapes and reasoning. Example: Examine whether all quadrilaterals have right angles. Give examples and non-examples. Example:  If the opposite sides on a parallelogram are parallel and congruent, then rectangles are parallelograms. A sample of questions that might be posed to students include: A parallelogram has 4 sides with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms?  Regular polygons have all of their sides and angles congruent. Name or draw some regular polygons. All rectangles have 4 right angles. Squares have 4 right angles so they are also rectangles. True or False? A trapezoid has 2 sides parallel so it must be a parallelogram. True or False?  •Explain and provide examples to show why attributes belonging to a category of two-dimensional shapes also belong to all subcategories of that category. http://illuminations.nctm.org/ActivityDetail.aspx?ID=70  **Common Core: 5.G.4**: Classify two-dimensional figures in a hierarchy based on properties.  **Unpacking:** This standard builds on what was done in 4th grade. Figures from previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle. Student should be able to reason about the attributes of shapes by examining: What are ways to classify triangles? Why can’t trapezoids and kites be classified as parallelograms? Which quadrilaterals have opposite angles congruent and why is this true of certain quadrilaterals?, and How many lines of symmetry does a regular polygon have?  Sort and classify two-dimensional figures in a hierarchy by identifying the main category that all shapes fit, and then separating the shapes into subcategories, and then if possible, separates shapes within the subcategories into another subcategory.  For example: | |

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| **M05.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.D-M.1 Convert like measurement units within a given measurement system. | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **conversion/convert, metric and customary measurement From previous grades: relative size, liquid volume, mass, length, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), hour, minute, second** | |
| **DESCRIPTOR**  **M05.D-M.1.1:** Solve problems using simple conversions (may include multistep, real-world problems). | **ELIGIBLE CONTENT**  **M05.D-M.1.1.1:** Convert among different-sized measurement units within a given  measurement system. A table of equivalencies will be provided.  Example: Convert 5 cm to meters. |
| **PA:CC.2.4.5.A.1:** Solve problems using conversions within a given measurement system.  **Common Core: 5.MD.1**: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm  to 0.05 m), and use these conversions in solving multi-step, real world problems.  **Unpacking:** 5.MD.1 calls for students to convert measurements within the same system of measurement in the context of multi-step, real-world problems. Both customary and standard measurement systems are included; students worked with both metric and customary units of length in second grade. In third grade, students work with metric units of mass and liquid volume. In fourth grade, students work with both systems and begin conversions within systems in length, mass and volume. Students should explore how the base-ten system supports conversions within the metric system. Example: 100 cm = 1 meter. | |

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| **M05.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.D-M.2 Represent and interpret data. | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **line plot, length, mass, liquid volume** | |
| **DESCRIPTOR**  **M05.D-M.2.1:** Organize, display, and answer questions based on data. | **ELIGIBLE CONTENT**  **M05.D-M.2.1.1:** Solve problems involving computation of fractions by using information presented in  line plots.  **M05.D-M.2.1.2:** Display and interpret data shown in tallies, tables, charts, pictographs, bar graphs,  and line graphs, and use a title, appropriate scale, and labels. A grid will be provided  to display data on bar graphs or line graphs. |
| **PA:CC.2.4.5.A.2:** Represent and interpret data using appropriate scale.  **PA:CC.2.4.5.A.4:** Solve problems involving computation of fractions using information provided in a line plot.  **Common Core: 5.MD.2**: Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*  **Unpacking:**  Measure several objects (at least 10 objects is suggested) to the nearest eighth, quarter or half, and then  • Construct a line plot with:  - A horizontal axis with tick marks or numbers that are evenly spaced out; - Enough tick marks or numbers to include the entire range of data;  - At least two tick marks numbered (usually, at least the first and last tick mark be numbered, but all tick marks can be numbered, or every other  tick mark can be numbered) - The axis labeled with a name and the unit of measurement; and  - X’s (or other symbol) that are a uniform size and vertically stacked uniformly.  • Use operations on fractions to solve problems involving information interpreted from the line plot.  Example: Ten beakers, measured in liters, are filled with a liquid. The line plot above shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.) Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers. | |

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| **M05.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M05.D-M.3 Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. | |
| Overview and vocabulary: Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measurement, attribute, volume, solid figure, right rectangular prism, unit, unit cube, gap, overlap, cubic units (cubic cm, cubic in. cubic ft. nonstandard cubic units), multiplication, addition, edge lengths, height, area of base** | |
| **DESCRIPTOR**  **M05.D-M.3.1**: Use, describe, and develop  procedures to solve problems involving volume. | **ELIGIBLE CONTENT**  **M05.D-M.3.1.1:** Apply the formulas V = l x w x h and V = B x h for rectangular prisms to find volumes  of right rectangular prisms with whole-number edge lengths in the context of solving  real-world and mathematical problems. Formulas will be provided.  **M05.D-M.3.1.2:** Find volumes of solid figures composed of two non-overlapping right rectangular  prisms. |
| **PA:CC.2.4.5.A.6:** Apply concepts of volume to solve problems and relate volume to multiplication and to addition.  **Common Core: 5.MD.5:** Relatevolume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.  a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the  same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold  whole-number products as volumes, e.g., to represent the associative property of multiplication.  b. Apply the formulas *V* = *l* . *w* . *h* and *V* = *B* . *h* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge  lengths in the context of solving real world and mathematical problems.  c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the  volumes of the non-overlapping parts, applying this technique to solve real world problems.  **Unpacking:**  • Fill a given rectangular prism with inch (or centimeter) tiles and report the volume of the rectangular prism as cubic inches (or cubic centimeters).  • Identify the length, width, and height of the same rectangular prism as above, multiply those measurements, and relate the product to the number of  inch (or centimeter) cubes that filled the rectangular prism.  • Explain how the volume of the rectangular prism can be determined when the length, width, and height are known.  • Solve real-world and mathematical problems involving the volume of rectangular prisms with whole-number side lengths.  • Decompose a figure that is made up of two or more rectangular prisms (not necessarily the same size) that are attached to each other, into separate r  rectangular prisms, and then find the volume of each separate prism and add them together to get the total volume of the original figure.    **5. MD.5a & b** - These standards involve finding the volume of right rectangular prisms (see picture above).Students should have experiences to describe and reason about why the formula is true. Specifically, that they are covering the bottom of a right rectangular prism (length x width) with multiple layers (height). Therefore, the formula (length x width x height) is an extension of the formula for the area of a rectangle.  **5.MD.5c** - This standard calls for students to extend their work with the area of composite figures into the context of volume. Students should be given concrete experiences of breaking apart (decomposing) 3-dimensional figures into right rectangular prisms in order to find the volume of the entire 3-dimensional figure.  Students need multiple opportunities to measure volume by filling rectangular prisms with cubes and looking at the relationship between the total volume and the area of the base. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units. | |

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| **M05.D-M Measurement and Data Reporting Category** |
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| ASSESSMENT ANCHOR  These “National” Common Core Standards do not have PA Common Core standards that correlate specifically. |
| **Common Core: 5.MD.3:** Recognize volume as an attribute of solid figures and understand concepts of volume measurement.  a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.  b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.  **Unpacking**: 5. MD.3, 5.MD.4, and 5. MD.5 These standards represent the first time that students begin exploring the concept of volume. In third grade, students begin working with area and covering spaces. The concept of volume should be extended from area with the idea that students are covering an area (the bottom of cube) with a layer of unit cubes and then adding layers of unit cubes on top of bottom layer (see picture below). Students should have ample experiences with concrete manipulatives before moving to pictorial representations. Students’ prior experiences with volume were restricted to liquid volume. As students develop their understanding volume they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in3, m3). Students connect this notation to their understanding of powers of 10 in our place value system. Models of cubic inches, centimeters, cubic feet, etc are helpful in developing an image of a cubic unit. Students estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.  **Common Core:5.MD.4:** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.  **Unpacking: •**Explain what it means to find the volume of something.  **•**Explain the proper process for using a packing/filling strategy for finding volume with unit cubes.  **•**Give examples of units that can be used when measuring volume. For example, if the unit cube is an inch cube, then the unit used is  “cubic inches.” If the unit cube is a centimeter cube, then the unit used is “cubic centimeters.”  • Determine the volume of a figure (such as a rectangular prism) by packing/filling it with “unit cubes” (without any gaps over overlaps), and then reporting the volume with the correct units. |

Table 1 Common addition and subtraction situations

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|  | **Result Unknown** | **Change Unknown** | **Start Unknown** |
| **Add to** | Two bunnies sat on the grass. Three  more bunnies hopped there. How many  bunnies are on the grass now?  2 + 3 = ? | Two bunnies were sitting on the grass.  Some more bunnies hopped there. Then  there were five bunnies. How many  bunnies hopped over to the first two?  2 + ? = 5 | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?  ? + 3 = 5 |
| **Take From** | Five apples were on the table. I ate two  apples. How many apples are on the  table now?  5 – 2 = ? | Five apples were on the table. I ate  some apples. Then there were three  apples. How many apples did I eat?  5 – ? = 3 | Some apples were on the table. I ate two  apples. Then there were three apples.  How many apples were on the table  before? ? – 2 = 3 |
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|  | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown2** |
| **Put Together/ Take Apart3** | Three red apples and two green apples  are on the table. How many apples are  on the table?  3 + 2 = ? | Five apples are on the table. Three are  red and the rest are green. How many  apples are green?  3 + ? = 5, 5 – 3 = ? | Grandma has five flowers. How many  can she put in her red vase and how  many in her blue vase?  5 = 0 + 5, 5 = 5 + 0  5 = 1 + 4, 5 = 4 + 1  5 = 2 + 3, 5 = 3 + 2 |
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|  | Difference Unknown | Bigger Unknown | Smaller Unknown |
| **Compare4** | (“How many more?” version):  Lucy has two apples. Julie has five  apples. How many more apples does  Julie have than Lucy?  (“How many fewer?” version):  Lucy has two apples. Julie has five  apples. How many fewer apples does  Lucy have than Julie?  2 + ? = 5, 5 – 2 = ? | (Version with “more”):  Julie has three more apples than Lucy.  Lucy has two apples. How many apples  does Julie have?  (Version with “fewer”):  Lucy has 3 fewer apples than Julie.  Lucy has two apples. How many apples  does Julie have?  2 + 3 = ?, 3 + 2 = ? | (Version with “more”):  Julie has three more apples than Lucy.  Julie has five apples. How many apples  does Lucy have?  (Version with “fewer”):  Lucy has 3 fewer apples than Julie.  Julie has five apples. How many apples  does Lucy have?  5 – 3 = ?, ? + 3 = 5 |

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2These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

3Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

4For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2 Common multiplication and division situations1

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|  | **Unknown Product**  **3 x 6 = ?** | **Group Size Unknown**  **(“How many in each group?” Division)**  **3 x ? = 18 and 18 ÷ 3 = ?** | **Number of Groups Unknown**  **(“How many groups?” Division)**  **? x 6 = 18 and 18 ÷ 6 = ?** |
| **Equal Groups** | There are 3 bags with 6 plums in  each bag. How many plums are  there in all?  *Measurement example.* You need 3  lengths of string, each 6 inches  long. How much string will you  need altogether? | If 18 plums are shared equally into 3 bags,  then how many plums will be in each bag?  *Measurement example.* You have 18 inches  of string, which you will cut into 3 equal  pieces. How long will each piece of string  be? | If 18 plums are to be packed 6 to a  bag, then how many bags are needed?  *Measurement example.* You have 18  inches of string, which you will cut  into pieces that are 6 inches long. How  many pieces of string will you have? |
| **Arrays2 , Area3** | There are 3 rows of apples with 6  apples in each row. How many  apples are there?  *Area example.* What is the area of a  3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows,  how many apples will be in each row?  *Area example.* A rectangle has area 18  square centimeters. If one side is 3 cm long,  how long is a side next to it? | If 18 apples are arranged into equal  rows of 6 apples, how many rows will  there be?  *Area example.* A rectangle has area 18  square centimeters. If one side is 6 cm  long, how long is a side next to it? |
|  |  |  |  |
|  | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown2** |
| **Compare** | A blue hat costs $6. A red hat costs  3 times as much as the blue hat.  How much does the red hat cost?  *Measurement example.* A rubber  band is 6 cm long. How long will  the rubber band be when it is  stretched to be 3 times as long? | A red hat costs $18 and that is 3 times as  much as a blue hat costs. How much does a  blue hat cost?  *Measurement example.* A rubber band is  stretched to be 18 cm long and that is 3  times as long as it was at first. How long  was the rubber band at first? | A red hat costs $18 and a blue hat  costs $6. How many times as much  does the red hat cost as the blue hat?  *Measurement example.* A rubber band  was 6 cm long at first. Now it is  stretched to be 18 cm long. How many  times as long is the rubber band now as  it was at first? |
| **General** | a x b = ? | a x ? = p and p ÷ a = ? | ? x b = p and p ÷ b = ? |

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1The first examples in each cell are examples of discrete things. These are easier or students and should be given before the measurement examples.

2The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in

the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

3Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially

important measurement situations.

Table 3 The properties of operations.

Here *a, b* and *c* stand for arbitrary numbers in a given number system. The properties apply to the rational number system, the real number system, and the complex number system.

|  |  |
| --- | --- |
| **Property** | **Example** |
| *Associative property of addition* | (a + b) + c = a + (b + c) |
| *Commutative property of addition* | a + b = b + a |
| *Additive identity property of 0* | a + 0 = 0 + a = a |
| *Associative property of multiplication* | (a x b) x c = a x (b x c) |
| *Commutative property of multiplication* | a x b = b x a |
| *Multiplicative identity property of 1* | a x 1 = 1 x a = a |
| *Distributive property of multiplication over addition* | a x (b + c) = a x b + a x c |