GRADE 3

Mathematics Assessment Anchors and Eligible content: Aligned to Pennsylvania Common Core Standards and to the Common Core State Standards for Mathematics (CCSS).

The Assessment Anchors, as defined by the Eligible Content, are organized into cohesive blueprints, each

structured with a common labeling system that can be read like an outline. This framework is organized first by

Reporting Category, then by Assessment Anchor, followed by Anchor Descriptor, and then finally, at the greatest level of detail, by an Eligible Content statement. The common format of this outline is followed across the PSSA.

Here is a description of each level in the labeling system for the PSSA:

**Reporting Category**

The Assessment Anchors are organized into four classifications, as listed below.

•A = Numbers and Operations •C = Geometry

•B = Algebraic Concepts •D = Data Analysis and Probability

These four classifications are used throughout the grade levels. In addition to these classifications, there are five Reporting Categories for each grade level. The first letter of each Reporting Category represents the classification; the second letter represents the Domain as stated in the Common Core State Standards for Mathematics. Listed below are the Reporting Categories for Grade 3.

•A-T = Number and Operations in Base Ten

•A-F = Number and Operations - Fractions

•B-O = Operations and Algebraic Thinking

•C-G = Geometry

•D-M = Measurement and Data

The title of each Reporting Category is consistent with the title of the corresponding Domain in the Common Core State Standards for Mathematics. The Reporting Category title appears at the top of each page.

**Assessment Anchor**

The Assessment Anchor appears in the shaded bar across the top of each Assessment Anchor table. The

Assessment Anchors represent categories of subject matter (skills and concepts) that anchor the content of the PSSA. Each Assessment Anchor is part of a Reporting Category and has one or more Anchor Descriptors unified under and aligned to it.

**Anchor Descriptor**

Below each Assessment Anchor is one or more specific Anchor Descriptors. The Anchor Descriptor adds a level of specificity to the content covered by the Assessment Anchor. Each Anchor Descriptor is part of an Assessment Anchor and has one or more Eligible Content unified under and aligned to it.

**Eligible Content**

The column to the right of the Anchor Descriptor contains the Eligible Content statements. The Eligible

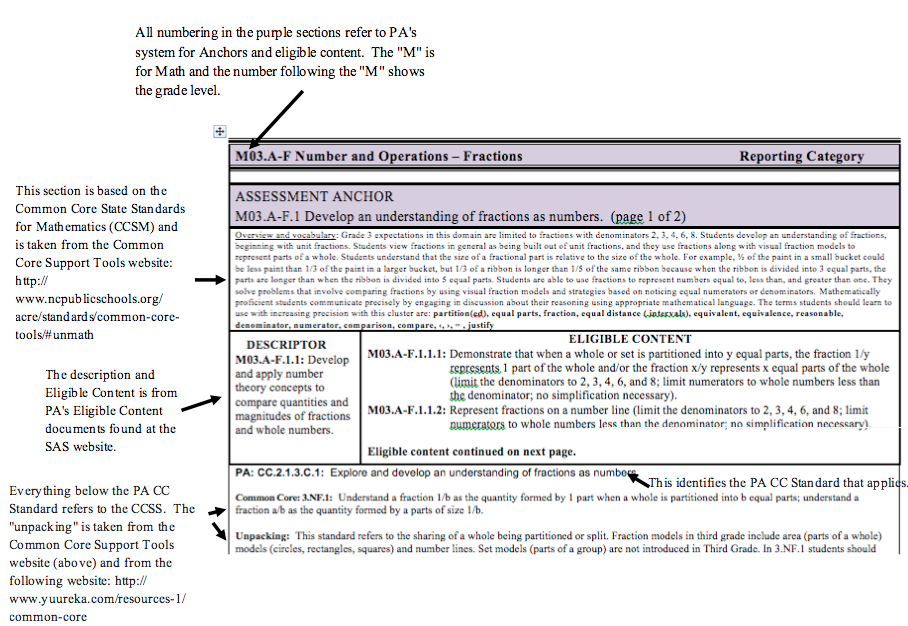
Content is the most specific description of the skills and concepts assessed on the PSSA. This level is considered the assessment limit and helps educators identify the range of the content covered on the PSSA. **Note:** All Grade 3 Eligible Content is considered Non-Calculator.

**Reference**

In the space below each Assessment Anchor table is a code representing one or more Common Core State Standards for Mathematics that correlate to the Eligible Content statements.

Alignment to the “National” Common Core and “unpacking” can be found below the PA CC Standard.

How Do I Read This Document?



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| **M03.A-T Number and Operations in Base Ten Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.A-T.1 Use place value understanding and properties of operations to perform multi-digit arithmetic. | |
| Overview and vocabulary: A range of algorithms may be used. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, estimation, round, addition, add, addend, sum, subtraction, subtract, difference, strategies, (properties) – rules about how numbers work** | |
| **DESCRIPTOR**  **M03.A-T.1.1:** Apply place value strategies to solve problems. | **ELIGIBLE CONTENT**  **M03.A-T.1.1.1:** Round two- and three-digit whole numbers to the nearest ten or  hundred, respectively.  **M03.A-T.1.1.2:** Add two- and three-digit whole numbers (limit sums from 100  through 1,000), and/or subtract two- and three-digit numbers from  three-digit whole numbers.  **M03.A-T.1.1.3:** Multiply one-digit whole numbers by two-digit multiples of 10  (from 10 through 90).  **M03.A-T.1.1.4:** Order a set of whole numbers from least to greatest or greatest to  least (up through 9,999; limit sets to no more than four numbers). |
| **PA CC.2.1.3.B.1:** Apply place value understanding and properties of operations to perform multi-digit arithmetic.  **Common Core: 3.NBT.1:** Use place value understanding to round whole numbers to the nearest 10 or 100.  **Unpacking:** This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.  **Common Core: 3.NBT.2:** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.  **Unpacking:** This standard refers to fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). Apply efficient strategies (especially mental strategies) to add and subtract within 1000 with minimal hesitation (no counting strategies should be used), after accumulating lots of experience with using a variety of strategies (including algorithms based on place value, such as stacking the numbers and adding the ones column first, and then the tens column, etc.) to add and subtract. The word algorithm refers to a procedure or a series of steps. There are other algorithms other than the standard algorithm. Third grade students should have experiences beyond the standard algorithm. A variety of algorithms will be assessed on EOG. Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable.  **Common Core: 3.NBT.3:** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.  **Unpacking:** This standard extends students’ work in multiplication by having them apply their understanding of place value.  This standard expects that students go beyond tricks that hinder understanding such as “just adding zeros” and explain and reason about their products. For example, for the problem 50 x 4, students should think of this as 4 groups of 5 tens or 20 tens. Twenty tens equals 200.    **NOTE:** Familiarity of single-digit multiplication facts is extremely helpful in doing this Standard. | |

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| **M03.A-F Number and Operations – Fractions**  **Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.A-F.1 Develop an understanding of fractions as numbers. (page 1 of 2) | |
| Overview and vocabulary: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8. Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, ½ of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **partition(ed), equal parts, fraction, equal distance ( intervals), equivalent, equivalence, reasonable, denominator, numerator, comparison, compare, ‹, ›, = , justify** | |
| **DESCRIPTOR**  **M03.A-F.1.1:** Develop and apply number theory concepts to compare quantities and magnitudes of fractions and whole numbers. | **ELIGIBLE CONTENT**  **M03.A-F.1.1.1:** Demonstrate that when a whole or set is partitioned into y equal parts, the fraction 1/y  represents 1 part of the whole and/or the fraction x/y represents x equal parts of the whole  (limit the denominators to 2, 3, 4, 6, and 8; limit numerators to whole numbers less than  the denominator; no simplification necessary).  **M03.A-F.1.1.2:** Represent fractions on a number line (limit the denominators to 2, 3, 4, 6, and 8; limit  numerators to whole numbers less than the denominator; no simplification necessary).  **Eligible content continued on next page.** |
| **PA: CC.2.1.3.C.1:**  Explore and develop an understanding of fractions as numbers.  **Common Core: 3.NF.1:** Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.  **Unpacking:** This standard refers to the sharing of a whole being partitioned or split. Fraction models in third grade include area (parts of a whole) models (circles, rectangles, squares) and number lines. Set models (parts of a group) are not introduced in Third Grade. In 3.NF.1 students should focus on the concept that a fraction is made up (composed) of many pieces of a unit fraction, which has a numerator of 1. For example, the fraction 3/5 is composed of 3 pieces that each have a size of 1/5.  Some important concepts related to developing understanding of fractions include: • Understand fractional parts must be equal-sized.  **Example** **Non-example**  • The number of equal parts tell how many make a whole. • As the number of equal pieces in the whole increases, the size of the fractional pieces decreases. • The size of the fractional part is relative to the whole.  • The number of children in one-half of a classroom is different than the number of children in one-half of a school. (the whole in each set is different therefore the half in each set will be different)  • When a whole is cut into equal parts, the denominator represents the number of equal parts.  • The numerator of a fraction is the count of the number of equal parts.  • 3/4 means that there are 3 one-fourths.  • Students can count *one fourth, two fourths, three fourths*.    These are thirds These are NOT thirds  **Common Core: 3.NF.2:** Understand a fraction as a number on the number line; represent fractions on a number line diagram.  a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts.  Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.  b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b  and that its endpoint locates the number a/b on the number line.  **Unpacking:** The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that ½ is between 0 and 1). In the number line diagram below, the space between 0 and 1 is divided (partitioned) into 4 equal regions. The distance from 0 to the first segment is 1 of the 4 segments from 0 to 1 or ¼ (3.NF.2a). Similarly, the distance from 0 to the third segment is 3 segments that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction ¾ (3.NF.2b).    Students express fractions as fair sharing, parts of a whole, and parts of a set. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require fair sharing.  THIS ANCHOR CONTINUED ON NEXT PAGE | |

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| **M03.A-F Number and Operations – Fractions**  **Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.A-F.1 Develop an understanding of fractions as numbers. (page 2 of 2) | |
| **DESCRIPTOR**  **M03.A-F.1.1:** Develop and apply number theory concepts to compare quantities and magnitudes of fractions and whole numbers. | **ELIGIBLE CONTENT**  **Continued from previous page (same anchor).**  **M03.A-F.1.1.3:** Recognize and generate simple equivalent fractions (limit the denominators to 1, 2, 3, 4, 6,  and 8; limit numerators to whole numbers less than the denominator).  Example 1: 1/2 = 2/4 Example 2: 4/6 = 2/3  **M03.A-F.1.1.4:** Express whole numbers as fractions, and/or generate fractions that are equivalent to whole  numbers (limit the denominators to, 2, 3, 4, 6, and 8).  Example 1: Express 3 in the form 3 = 3/1. Example 2: Recognize that 6/1 = 6.  **M03.A-F.1.1.5:** Compare two fractions with the same denominator (limit the denominators to 1, 2, 3, 4, 6,  and 8), using the symbols >, =, or and/or justify the conclusions. |
| **PA: CC.2.1.3.C.1:**  Explore and develop an understanding of fractions as numbers.  **Common Core: 3.NF.3**: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.  a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.  b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual  fraction model.  c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1;*  *recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*  d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid  only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the  conclusions, e.g., by using a visual fraction model.  **Unpacking:** Recognize when two fractions are equivalent, and explain why they are equivalent.  •Generate simple equivalent fractions (or can determine a fraction that is equivalent to a given fraction).  •Relate 1 to any fraction *a*/*a* (and vice versa), and explain why they are equivalent.  •Express whole numbers as fractions (such as 3 = 3/1) and explain why they are equivalent.  •Compare two fractions with the same denominator by reasoning that both fractions are partitioned equally and so the fraction with the larger numerator is larger because it has more equal parts that the other.  •Compare two fractions with the same numerator by reasoning that both fractions require the same number of parts (as indicated by the numerator) and a larger denominator means that the whole is partitioned into more (and smaller) equal parts.  •Use the symbols >, =, or <, to record how two fractions compare, and explain.  An important concept when comparing fractions is to reason about the size of the parts and the number of the parts. For example, 1/8 is smaller than  1/2 because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.  **3.NF.3a and 3.NF.3b** These standards call for students to use visual fraction models (area models) and number lines to explore the idea of equivalent fractions. Students should only explore equivalent fractions using models, rather than using algorithms or procedures.  **3.NF.3c** This standard includes writing whole numbers as fractions. The concept relates to fractions as division problems, where the fraction 3/1 is 3 wholes divided into one group. This standard is the building block for later work where students divide a set of objects into a specific number of groups. Students must understand the meaning of a/1. Example: If 6 brownies are shared between 2 people, how many brownies would each person get? | |

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| **M03.B-O Operations and Algebraic Thinking Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.B-O.1 Represent and solve problems involving multiplication and division. | |
| Overview and vocabulary: Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **products, groups of, quotients, partitioned equally, multiplication, division, equal groups, arrays, patterns, equations, unknown** | |
| **DESCRIPTOR**  **M03.B-O.1.1:** Understand various meanings of multiplication and division. | **ELIGIBLE CONTENT**  **M03.B-O.1.1.1:** Interpret and/or describe products of whole numbers (up to and  including 10 x 10). Example 1: Interpret 35 as the total number of  objects in 5 groups, each containing 7 objects. Example 2: Describe  a context in which a total number of objects can be expressed as 5x7.  **M03.B-O.1.1.2:** Interpret and/or describe whole-number quotients of whole numbers  (limit dividends through 50, and limit divisors and quotients through  10). Example 1: Interpret 48 ÷ 8 as the number of objects in each  share when 48 objects are partitioned equally into 8 shares, or as a  number of shares when 48 objects are partitioned into equal shares  of 8 objects each Example 2: Describe a context in which a number  of shares or a number of groups can be expressed as 48 ÷ 8. |
| **PA: CC.2.2.3.A.1:** Represent and solve problems involving multiplication and division.  **Common Core: 3.OA.1:** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.  **Unpacking:** Describe a context for the product of two numbers. For example, for the product of *m* x *n*, the student might describe the context as the:  • total number of objects in *m* groups/sets of *n* objects  • total number of objects in *m* rows/columns of *n* objects;  • total number of objects in an *m* . *n* rectangular array;  • number of square units (the area) in an *m* . *n* rectangle;  • total distance on a number line after *m* jumps of *n* units per jump;  • total length when *m* units are repeated *n* times.  **Common Core: 3.OA.2:** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.  For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.  **Unpacking:** This standard focuses on two distinct models of division: partition models and measurement (repeated subtraction) models.  Partition models focus on the question, “How many in each group?” A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag?    Measurement (repeated subtraction) models focus on the question, “How many groups can you make?” A context for measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill? | |

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| **M03.B-O Operations and Algebraic Thinking Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.B-O.1 Represent and solve problems involving multiplication and division. | |
| **DESCRIPTOR**  **M03.B-O.1.2** Solve mathematical and real- world problems using multiplication and division, including determining the missing number in a multiplication and/or division equation. | **ELIGIBLE CONTENT**  **M03.B-O.1.2.1:** Use multiplication (up to and including 10 x 10) and/or division  (limit dividends through 50, and limit divisors and quotients through  10) to solve word problems in situations involving equal groups,  arrays, and/or measurement quantities.  **M03.B-O.1.2.2:** Determine the unknown whole number in a multiplication (up to and  including 10 x 10) or division (limit dividends through 50, and limit  divisors and quotients through 10) equation relating three whole  numbers. Example: Determine the unknown number that makes an  equation true. |
| **PA: CC.2.2.3.A.1:** Represent and solve problems involving multiplication and division.  **Common Core: 3.OA.3:** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (Note: See Appendix, Table 2.)  **Unpacking:** When presented with word problems involving multiplication or division within 100 that are presented in situations involving equal groups, arrays, or measurement quantities: •Represent the problem with equation using a symbol (such as a blank or empty box or question mark) to represent the unknown value; •Use objects or drawings that appropriately models the situation described in the problem; and •Find the correct solution (i.e., the missing number).   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  | |  |  |  |  |  |  |   This standard references various strategies that can be used to solve word problems involving multiplication & division. Students should apply their skills to solve word problems. Students should use a variety of representations for creating and solving one-step word problems, such as: If you divide 4 packs of 9 brownies among 6 people, how many cookies does each person receive? **(4 x 9 = 36, 36 ÷ 6 = 6).** Glossary page 89, Table 2 gives examples of a variety of problem solving contexts, in which students need to find the product, the group size, or the number of groups. Students should be given ample experiences to explore all of the different problem structures. Examples of multiplication: There are 24 desks in the classroom. If the teacher puts 6 desks in each row, how many rows are there? This task can be solved by drawing an array by putting 6 desks in each row. This is an array model:    This task can also be solved by drawing pictures of equal groups.4 groups of 6 equals 24 objects:    A student could also reason through the problem mentally or verbally, “I know 6 and 6 are 12. 12 and 12 are 24. Therefore, there are 4 groups of 6 giving a total of 24 desks in the classroom.” A number line could also be used to show equal jumps. Students in third grade students should use a variety of pictures, such as stars, boxes, flowers to represent unknown numbers (variables).. Letters are also introduced to represent unknowns in third grade. Examples of Division: There are some students at recess. The teacher divides the class into 4 lines with 6 students in each line. Write a  division equation for this story and determine how many students are in the class **(􀀀 ÷ 4 = 6. There are 24 students in the class).**  **Common Core: 3.OA.4:** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 =  ÷ 3, 6 × 6 = ?.  **Unpacking:**  Determine the missing value in multiplication and division problems that resemble these equations:  • m × n = \_\_\_ \_\_\_ × n = p m × \_\_ = p (where m, n, and p are given values)  • m ÷ n = \_\_\_ \_\_\_ ÷ n = q m ÷ \_\_ = q (where m, n, and q are given values).  **NOTE**: The unknown value can be represented by a blank, empty box, question mark, or some other symbol.  The focus of **3.OA.4** goes beyond the traditional notion of fact families, by having students explore the inverse relationship of multiplication and division. Students apply their understanding of the meaning of the equal sign as ”the same as” to interpret an equation with an unknown. When given 4 x ? = 40, they might think:  • 4 groups of some number is the same as 40  • 4 times some number is the same as 40  • I know that 4 groups of 10 is 40 so the unknown number is 10  • The missing factor is 10 because 4 times 10 equals 40. | |

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| **M03.B-O Operations and Algebraic Thinking Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.B-O.2 Understand properties of multiplication and the relationship between multiplication and division. | |
| Overview and vocabulary: Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operation, multiply, divide, factor, product, quotient, strategies, (properties)-rules about how numbers work** | |
| **DESCRIPTOR**  **M03.B-O.2.1:** Use properties to simplify and solve multiplication problems. | **ELIGIBLE CONTENT**  **M03.B-O.2.1.1:** Apply the commutative property of multiplication (not identification  or definition of the property).  **M03.B-O.2.1.2:** Apply the associative property of multiplication (not identification  or definition of the property). |
| **PA:CC.2.2.3.A.2:** Understand properties of multiplication and the relationship between multiplication and division.  **Common Core: 3.OA.5:** Apply properties of operations as strategies to multiply and divide. (Note: Students need not use formal terms for these properties.) Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56. (Distributive property.)  **Unpacking:**  Give the answer to m × n if he already knows n × m and can explain that he applied the commutative property (it is not necessary to use formal term of commutative property, but can explain the essence of the property).  • Apply the associative and commutative properties to:  • strategically multiply a string of numbers (such as 4 × 3 × 5), especially recognizing if any factors multiple to make 10 or a multiple of 10 or  other benchmark numbers that are easier to multiply (e.g., 4 × 3 × 5 = 4 × 5 × 3 = 20 × 3 = 60);  • decompose a number into smaller factors to make it easier to multiply (e.g., if the student has difficulty multiplying 8 × 6, the student  decomposes the 6 to get 8 × 3 × 2, and does 8 × 3 first and then multiplies 24 by 2.  • Apply the distributive property to deconstruct numbers that might be difficult to multiply (for example, if student has difficulty multiplying  8 × 6, the student might write 8 × (5 + 1) = (8 × 5) + (8 × 1) = 40 + 8 = 48.  While students DO NOT need to not use the formal terms of these properties, student should understand that properties are rules about how numbers work, they need to be flexibly and fluently applying each of them. Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication.  The array below could be described as a 5 x 4 array for 5 columns and 4 rows, or a 4 x 5 array for 4 rows and 5 columns. There is no “fixed” way to write the dimensions of an array as rows x columns or columns x rows.   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  |   To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations are true or false.  • 0 x 7 = 7 x 0 = 0 (Zero Property of Multiplication)  • 1 x 9 = 9 x 1 = 9 (Multiplicative Identity Property of 1)  • 3 x 6 = 6 x 3 (Commutative Property)  • 8 ÷ 2 = 2 ÷ 8 (Students are only to determine that these are not equal)  • 2 x 3 x 5 = 6 x 5  • 10 x 2 < 5 x 2 x 2  • 2 x 3 x 5 = 10 x 3  • 0 x 6 > 3 x 0 x 2 | |

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| **M03.B-O Operations and Algebraic Thinking Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.B-O.2 Understand properties of multiplication and the relationship between multiplication and division. | |
| **DESCRIPTOR**  **M03.B-O.2.2:** Relate division to a missing number multiplication equation. | **ELIGIBLE CONTENT**  **M03.B-O.2.2.1:** Interpret and/or model division as a multiplication equation with an  unknown factor. Example: Find 32 ÷ 8 by solving 8 x ? = 32. |
| **PA:CC.2.2.3.A.2:** Understand properties of multiplication and the relationship between multiplication and division.  **Common Core: 3.OA.6:** Understand division as an unknown-factor problem. EX: find 32÷8 by finding the number that makes 32 when multiplied by 8.  **Unpacking:** • Take a division equation (in the form *m* ÷ *n* = \_\_\_, where are *m* is a whole number divisible by *n*) and rewrite it as a multiplication problem with a missing factor (in the form \_\_\_ x *n* = *m*, or *n* x \_\_\_ = *m*. •Explain how *m* ÷ *n* = \_\_\_ and *n* x \_\_\_ = *m* are related.  This standard refers the Glossary on page 89, Table 2 and the various problem structures. Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems. Example: A student knows that 2 x 9 = 18. How can they use that fact to determine the answer to the following question: 18 people are divided into pairs in P.E. class? How many pairs are there? Write a division equation and explain your reasoning.  Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient.  Examples: • 3 x 5 = 15 5 x 3 = 15 • 15 ÷ 3 = 5 15 ÷ 5 = 3 | |

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| **M03.B-O Operations and Algebraic Thinking Reporting Category** | |
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| PA:CC.2.2.3.A.3: Demonstrate multiplication and division fluency / Multiply and Divide within 100 | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operation, multiply, divide, factor, product, quotient, unknown, strategies, reasonableness, mental computation, property** | |
| **DESCRIPTOR**  N/A | **ELIGIBLE CONTENT**  Although demonstrating multiplication and division fluency is a PA CC Standard, it is not given a specific Eligible Content Statement. |
| **PA:CC.2.2.3.A.3:** Demonstrate multiplication and division fluency.  **Common Core: 3.OA.7:** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.  **Unpacking:** This standard uses the word fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). “Know from memory” should not focus only on timed tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9 x 9). By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Strategies students may use to attain fluency include: • Multiplication by zeros and ones • Doubles (2s facts), Doubling twice (4s), Doubling three times (8s) • Tens facts (relating to place value, 5 x 10 is 5 tens or 50) • Five facts (half of tens) • Skip counting (counting groups of \_\_ and knowing how many groups have been counted) • Square numbers (ex: 3 x 3) • Nines (10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3) • Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6) • Turn-around facts (Commutative Property)  • Fact families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24) • Missing factors  **NOTE:** Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.  **NOTE**: Memorizing multiplication facts from flash cards should not be sole or primary mode of learning the facts. Memorization of most of the multiplication facts come as a by-product of developing number sense after having worked with representations of multiplication | |

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| **M03.B-O Operations and Algebraic Thinking Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.B-O.3 Solve problems involving the four operations, and identify and explain patterns in arithmetic. | |
| **DESCRIPTOR**  **M03.B-O.3.1:** Use operations, patterns, and estimation strategies to solve problems (may include word problems). | **ELIGIBLE CONTENT**  **M03.B-O.3.1.1:** Solve two-step word problems using the four operations (expressions not explicitly stated).  Limit to problems with whole numbers and having whole-number answers.  **M03.B-O.3.1.2:** Represent two-step word problems using equations with a symbol standing for the unknown  quantity. Limit to problems with whole numbers and having whole-number answers.  **M03.B-O.3.1.3:** Assess the reasonableness of answers. Limit problems posed with whole numbers and  having whole-number answers.  **M03.B-O.3.1.4:** Solve two-step equations using order of operations (equation is explicitly stated with no  grouping symbols).  **M03.B-O.3.1.5:** Identify arithmetic patterns (including patterns in the addition table multiplication table)  and/or explain them using properties of operations. Example 1: Observe that 4 times a  number is always even. Example 2: Explain why 6 times a number can be decomposed into  three equal addends.  **M03.B-O.3.1.6:** Create or match a story to a given combination of symbols (+, –, x, ÷, <, >, =) and numbers.  **M03.B-O.3.1.7:** Identify the missing symbol (+, –, x, ÷, <, >, =) that makes a number sentence true. |
| **PA:CC.2.2.3.A.4:** Solve problems involving the four operations, and identify and explain patterns in arithmetic.  **Common Core: 3.OA.8:** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.3  3 This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.  **Unpacking:** This standard refers to two-step word problems using the four operations. The size of the numbers should be limited to related 3rd grade standards (e.g., 3.OA.7 and 3.NBT.2). Adding and subtracting numbers should include numbers within 1,000, and multiplying and dividing numbers should include single-digit factors and products less than 100. This standard calls for students to represent problems using equations with a letter to represent unknown quantities. Example: Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution (2 x 5 + m = 25). This standard refers to estimation strategies, including using compatible numbers (numbers that sum to 10, 50, or 100) or rounding. The focus in this standard is to have students use and discuss various strategies. Students should use the context of the problem, and use mental computation or estimation strategies to check if the answer is reasonable. Students should estimate during problem solving, and then revisit their estimate to check for reasonableness.  Example: Here are some typical estimation strategies for the problem: On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?  **Student 1:** I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500. **Student 2:** I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500. **Student 3:** I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530**.**  The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.  **NOTE**: Two-step word problems are solved by performing one computation and using that result as part of a second computation.  **Common Core: 3.OA.9:** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.  **Unpacking:** When given a set of numbers (such as on a hundreds chart, in an addition table or multiplication table, or a calendar of a month):  •Identify a pattern and explain why the pattern works. This standard calls for students to examine arithmetic patterns involving both addition and multiplication. Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term. This standards also mentions identifying patterns related to the properties of operations.  Examples: • Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends (14 = 7 + 7). • Multiples of even numbers (2, 4, 6, and 8) are always even numbers. • On a multiplication chart, the products in each row and column increase by the same amount (skip counting). • On an addition chart, the sums in each row and column increase by the same amount.  Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically. Example:• Any sum of two even numbers is even. • Any sum of two odd numbers is even.  • Any sum of an even number and an odd number is odd. • The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines. • The multiples of any number fall on a horizontal and a vertical line due to the commutative property. • All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a  multiple of 10. Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense. | |

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| **M03.C-G Geometry Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.C-G.1 Reason with shapes and their attributes. | |
| Overview and vocabulary: Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attributes, properties, quadrilateral, open figure, closed figure , three sided, 2-dimensional, 3-dimensional, rhombi, rectangles, and squares are subcategories of quadrilaterals, cubes, cones, cylinders, and rectangular prisms are subcategories of 3-dimensional figures shapes: polygon, rhombus/rhombi, rectangle, square, partition, unit fraction From previous grades: triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere** | |
| **DESCRIPTOR**  **M03.C-G.1.1:** Analyze characteristics of polygons. | **ELIGIBLE CONTENT**  **M03.C-G.1.1.1:** Explain that shapes in different categories may share attributes, and that the shared  attributes can define a larger category. Example 1: A rhombus and a rectangle are both  quadrilaterals since they both have exactly four sides. Example 2: A triangle and a pentagon  are both polygons since they are both multi-sided plane figures.  **M03.C-G.1.1.2:** Recognize rhombi, rectangles, and squares as examples of quadrilaterals, and/or draw  examples of quadrilaterals that do not belong to any of these subcategories.  **M03.C-G.1.1.3:** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction  of the whole. Example 1: Partition a shape into 4 parts with equal areas. Example 2:  Describe the area of each of 8 equal parts as 1/8 of the area of the shape. |
| **PA:CC.2.3.3.A.1:** Identify, compare, and classify shapes and their attributes.  **PA:CC.2.3.3.A.2:** Use the understanding of fractions to partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole.  **Common Core: 3.G.1:** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.  **Unpacking:** • Categorize shapes by their attributes; •Analyze the shapes in one category (such as “has 4 sides”) and sub-categorize those shapes by their attributes (e.g., the category of “has 4 sides” could have two sub-categories—“opposite sides are parallel” and “opposite sides are not parallel”);  • Draw examples that don’t fit into any of the categories specified.  In second grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. Third graders build on this experience and further investigate quadrilaterals (technology may be used during this exploration). Students recognize shapes that are and are not quadrilaterals by examining the properties of the geometric figures. They conceptualize that a quadrilateral must be a closed figure with four straight sides and begin to notice characteristics of the angles and the relationship between opposite sides. Students should be encouraged to provide details and use proper vocabulary when describing the properties of quadrilaterals. They sort geometric figures (see examples below) and identify squares, rectangles, and rhombuses as quadrilaterals.    Students should classify shapes by attributes and drawing shapes that fit specific categories. For example, parallelograms include: squares, rectangles, rhombi, or other shapes that have two pairs of parallel sides. Also, the broad category quadrilaterals include all types of parallelograms, trapezoids and other four-sided figures. Example: Draw a picture of a quadrilateral. Draw a picture of a rhombus. How are they alike? How are they different? Is a quadrilateral a rhombus? Is a rhombus a quadrilateral? Justify your thinking.  **Common Core: 3.G.2**: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*  **Unpacking:** • Partition a given shape into equal parts and label each part as a unit fraction of a whole. Examples of partitioning include the following: •Folding paper cut-outs; •Filling a shape with the same size parts; •Drawing in lines to partition the shape (especially when the shape is given on grid paper)  This standard builds on students’ work with fractions and area. Students are responsible for partitioning shapes into halves, thirds, fourths, sixths and eighths. Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways. | |

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| **M03.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.D-M.1 Solve problems involving measurement and estimation of intervals of time, money\*, liquid  volumes, masses, and lengths of objects. \*money not included in “National” CC at Grade 3 | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **estimate, time, time intervals, minute, hour, elapsed time, measure** | |
| **DESCRIPTOR**  **M03.D-M.1.1:** Determine or calculate time and elapsed time. | **ELIGIBLE CONTENT**  **M03.D-M.1.1.1:** Tell, show, and/or write time (analog) to the nearest minute.  **M03.D-M.1.1.2:** Calculate elapsed time to the minute in a given situation (total elapsed time limited to 60  minutes or less). |
| **PA:CC.2.4.3.A.2:** Tell and write time to the nearest minute and solve problems by calculating time intervals.  **Common Core: 3.MD.1**: Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.  **Unpacking:**  •Tell time to the nearest minute on an analog clock.  •Measure how many minutes have elapsed in an observed event.  •Determine the time if the student knows the start time and the number of minutes that have elapsed.  •Determine what time it was if the student knows the end time and the number of minutes that have elapsed.  •Determine how many minutes have elapsed if the student knows the start and end times.  This standard calls for students to solve elapsed time, including word problems. Students could use clock models or number lines to solve. On the number line, students should be given the opportunities to determine the intervals and size of jumps on their number line. Students could use pre-determined number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students).  Example:  Tonya wakes up at 6:45 a.m. It takes her 5 minutes to shower, 15 minutes to get dressed, and 15 minutes to eat breakfast. What time will she be ready for school? | |

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| **M03.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.D-M.1 Solve problems involving measurement and estimation of intervals of time, money\*, liquid  volumes, masses, and lengths of objects. \*money not included in “National” CC at Grade 3 | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, liquid volume, mass, standard units, metric, gram (g), kilogram (kg), liter (L), cup (c), pint (pt) quart (qt), gallon (gal), ounce (oz), and pounds (lb)** | |
| **DESCRIPTOR**  **M03.D-M.1.2:** Use the attributes of liquid volume, mass, and length of objects. | **ELIGIBLE CONTENT**  **M03.D-M.1.2.1:** Measure and estimate liquid volumes and masses of objects using standard units (cups  [c], pints [pt], quarts [qt], gallons [gal], ounces [oz.], and pounds [lb]) and metric units  (liters [l], grams [g], and kilograms [kg]).  **M03.D-M.1.2.2:** Add, subtract, multiply, and divide to solve one-step word problems involving masses or  liquid volumes that are given in the same units.  **M03.D-M.1.2.3:** Use a ruler to measure lengths to the nearest quarter inch or centimeter. |
| **PA:CC.2.4.3.A.1:** Solve problems involving measurement and estimation of temperature, liquid volume, mass or length.  **Common Core: 3.MD.2:** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Note: Excludes compound units such as cm3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Note: Excludes multiplicative comparison problems -- problems involving notions of “times as much”; see Appendix, Table 2.)  **Unpacking:** •Use the appropriate tools to measure liquid volumes in units of liters. •Estimate about how much 1 liter (or some other amount) is and can say whether or not a given amount of liquid is less than a liter, about a liter, or more than a liter. •Use appropriate tools to measure mass of objects in units of grams and kilograms. •Estimate the mass of objects in comparison to benchmark measurements (e.g., if student knows that his weight is 40 kilograms, he can estimate if something is less than his mass, about equal to his mass, or greater than is mass).  •Solve one-step measurement problems that involve adding, subtracting, multiplying, or dividing measurements of the same units.  This standard asks for students to reason about the units of mass and volume. Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter. Word problems should only be one-step and include the same units. Example: Students identify 5 things that weigh about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.) This activity helps develop gram benchmarks. One large paperclip weighs about one gram. A box of large paperclips (100 clips) weighs about 100 grams so 10 boxes would weigh one kilogram. Example: A paper clip weighs about a) a gram, b) 10 grams, c) 100 grams?  Foundational understandings to help with measure concepts:  •Understand that larger units can be subdivided into equivalent units (partition).  •Understand that the same unit can be repeated to determine the measure (iteration).  •Understand the relationship between the size of a unit and the number of units needed (compensatory principal).  **Common Core: 3.MD.4:** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.  **Unpacking:** Students in second grade measured length in whole units using both metric and U.S. customary systems. It’s important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment. This standard provides a context for students to work with fractions by measuring objects to a quarter of an inch. | |

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| **M03.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.D-M.1 Solve problems involving measurement and estimation of intervals of time, money\*, liquid  volumes, masses, and lengths of objects. \*money not included in “National” CC at Grade 3 | |
| **DESCRIPTOR**  **M03.D-M.1.3:** Count, compare, and make change using a collection of coins and one-dollar bills. | **ELIGIBLE CONTENT**  **M03.D-M.1.3.1:** Compare total values of combinations of coins (penny, nickel, dime, quarter) and/or  dollar bills less than $5.00.  **M03.D-M.1.3.2:** Make change for an amount up to $5.00 with no more than $2.00 change given (penny,  nickel, dime, quarter, and dollar).  **M03.D-M.1.3.3:** Round amounts of money to the nearest dollar. |
| **PA:CC.2.4.3.A.3:** Solve problems involving money using a combination of coins and bills.  **Common Core: Working with money appears in Grade 2 and Grade 4 in the “National” Common Core but does not appear in Grade 3.** | |

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| **M03.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.D-M.2 Represent and interpret data. (page 1 of 2) | |
| Overview and vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale, scaled picture graph, scaled bar graph, line plot, data** | |
| **DESCRIPTOR**  **M03.D-M.2.1:** Organize, display, and answer questions based on data. | **ELIGIBLE CONTENT**  **M03.D-M.2.1.1:** Complete a scaled pictograph and a scaled bar graph to represent a data set with several  categories (scales limited to 1, 2, 5, and 10).  **M03.D-M.2.1.2:** Solve one- and two-step problems using information to interpret data presented in scaled  pictographs and scaled bar graphs (scales limited to 1, 2, 5, and 10). Example 1: (One-  step) “Which category is the largest?” Example 2: (Two-step) “How many more are in  category A than in category B?” |
| **PA:CC.2.4.3.A.4:** Represent and interpret data using tally charts, tables, pictographs, line plots, and bar graphs.  **Common Core: 3.MD.3**: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*  **Unpacking**: \*For a data set with several categories, draw a picture graph (also known as a pictograph), with…  •A horizontal or vertical axis with the names of the categories labeled;  •A picture used to represent each data point and the picture is uniform in size and uniformly stacked; and  •A scale to show the quantity that one picture represents.  \*For a data set with up several categories, draw a bar graph with…  •A horizontal axis labeled with the names of the categories and a vertical axis with enough numbered tick marks to represent the  frequency of data points for each category (or a vertical axis labeled with the names of the categories and a horizontal axis with  tick marks to represent the frequency of data points for each category) [NOTE: the frequency axis need not be labeled by ones;  in fact, students should scale the axis by 2’s 5’s, 10’s or some other scale];  •Bars of uniform width drawn to the correct height/length to represent the frequency of data points for each category.  NOTE: Students should have an opportunity to construct the graph from scratch instead of being given a template.  Students should have opportunities reading and solving problems using scaled graphs before being asked to draw one. The following graphs all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts. While exploring data concepts, students should Pose a question, Collect data, Analyze data, and Interpret data (PCAI). Students should be graphing data that is relevant to their lives Example: Pose a question: Student should come up with a question. What is the typical genre read in our class?  Collect and organize data: student survey Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?    Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.  THIS ANCHOR CONTINUED ON NEXT PAGE.  Analyze and Interpret data:  • How many more nofiction books where read than fantasy books?  • Did more people read biography and mystery books or fiction and fantasy  books?  • About how many books in all genres were read?  • Using the data from the graphs, what type of book was read more often than  a mystery but less often than a fairytale?  • What interval was used for this scale?  • What can we say about types of books read? What is a typical type of book  read?  • If you were to purchase a book for the class library which would be the best  genre?Why? | |

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| **M03.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.D-M.2 Represent and interpret data. (page 2 of 2) | |
| **DESCRIPTOR**  **M03.D-M.2.1:** Organize, display, and answer questions based on data. | **ELIGIBLE CONTENT**  **M03.D-M.2.1.3:** Generate measurement data by measuring lengths using rulers marked with halves and  fourths of an inch. Display the data by making a line plot, where the horizontal scale is  marked in appropriate units—whole numbers, halves, or quarters.  **M03.D-M.2.1.4:** Translate information from one type of display to another. Limit to pictographs, tally  charts, bar graphs, and tables. Example: Convert a tally chart to a bar graph. |
| **PA:CC.2.4.3.A.4:** Represent and interpret data using tally charts, tables, pictographs, line plots, and bar graphs.  **Common Core: 3.MD.4:** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.  **Unpacking**:  Measure several objects (at least 10 objects is suggested) to the nearest quarter or half, and then  •Construct a line plot with…  •a horizontal axis with tick marks or numbers that are evenly spaced out;  •enough tick marks or numbers to include the entire range of data;  •at least two tick marks numbered (usually, at least the first and last tick mark be numbered, but all tick marks can be numbered, or  every other tick mark can be numbered);  •the axis labeled with a name and the unit of measurement; and  •X’s (or other symbol) that are a uniform size and vertically stacked uniformly.  Students in second grade measured length in whole units using both metric and U.S. customary systems. It’s important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment.  This standard provides a context for students to work with fractions by measuring objects to a quarter of an inch.  Example:  Measure objects in your desk to the nearest ½ or ¼ of an inch, display data collected on a line plot. How many objects measured ¼ ? ½ ? etc… | |

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| **M03.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.D-M.3 Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (page 1 of 2) | |
| Overview and vocabulary: Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, area, square unit, plane figure, gap, overlap, square cm, square m , square in., square ft, nonstandard units, tiling, side length, decomposing** | |
| **DESCRIPTOR**  **M03.D-M.3.1:**  Find the areas of plane figures. | **ELIGIBLE CONTENT**  **M03.D-M.3.1.1:** Measure areas by counting unit squares (square cm, square m, square in., square ft, and  non-standard square units).  **M03.D-M.3.1.2:** Multiply side lengths to find areas of rectangles with whole-number side lengths in the  context of solving real-world and mathematical problems, and represent whole-number  products as rectangular areas in mathematical reasoning. |
| **PA:CC.2.4.3.A.5:** Determine the area of a rectangle and apply the concept to multiplication and to addition.  **Common Core: 3.MD.5:** Recognize area as an attribute of plane figures and understand concepts of area measurement.  a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.  b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.  **Unpacking**: •Explain what it means to find the area of something.  •Explain the proper process for using a covering strategy for finding area with unit squares.  •Give examples of units that can be used when measuring area. For example, if the unit square is an inch square, then the unit used is  “square inches.” If the unit square is a centimeter square, then the unit used is “square centimeters.”  **Common Core: 3.MD.6:** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). Students should be counting the square units to find the area could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches.  **Unpacking:** Students should be counting the square units to find the area could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches.  Determine the area of a plane figure (especially squares and rectangles) by covering it with “unit squares” (without any gaps over overlaps), and then reporting the area with the correct units.  Common Core 3.MD.7 is also aligned with this Assessment Anchor. It is listed and unpacked on the next page. | |

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| **M03.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.D-M.3 Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (page 2 of 2) | |
| **DESCRIPTOR**  **M03.D-M.3.1:**  Find the areas of plane figures. | **ELIGIBLE CONTENT**  **M03.D-M.3.1.1:** Measure areas by counting unit squares (sq cm, sq m, sq in., sq ft, and non-standard square units).  **M03.D-M.3.1.2:** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving  real-world and mathematical problems, and represent whole-number products as rectangular areas in  mathematical reasoning. |

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| **PA:CC.2.4.3.A.5:** Determine the area of a rectangle and apply the concept to multiplication and to addition.  **Continued from previous page. Same PA Assessment Anchor, Descriptor, and Eligible Content**  **Common Core:** **3.MD.7**: Relate area to the operations of multiplication and addition.  a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by  multiplying the side lengths.  b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical  problems, and represent whole-number products as rectangular areas in mathematical reasoning.  c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* . *b* and *a* . *c*.  Use area models to represent the distributive property in mathematical reasoning.  d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of  the non-overlapping parts, applying this technique to solve real world problems.  **Unpacking:** Students should tile rectangle then multiply the side lengths to show it is the same.    **3.MD.7.a**  To find the area one could count the squares or multiply 3 x 4 = 12.      4 x 2 = 8 and  2 x 2 = 4 So 8 + 4 = 12  Therefore the total area of this figure is 12 square units.  +   |  |  | | --- | --- | |  |  | |  |  | |  |  | |  |  |     2 x 4   |  |  | | --- | --- | |  |  | |  |  |   2 x 2  **3.MD.7.d**  This standard uses the word rectilinear. A rectilinear figure is a polygon that has all right angles. How could the figure below be decomposed to help find the area?  **3.MD.7.c**  This standard extends students’ work with the distributive property. The first example below reflects one way to represent the distributive property. In the second example below the picture shows that the area of a  7 x 6 figure can be determined by finding the area of a 5 x 6 and 2 x 6 and adding the two sums.                **3MD.7.b**  Students should solve real world and mathematical problems Example:  Drew wants to tile the bathroom floor using 1foot tiles. How many square foot tiles will he need?     |  |  |  | | --- | --- | --- | | 1 | 2 | 3 | | 4 | 5 | 6 | | 7 | 8 | 9 | | 10 | 11 | 12 | |

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| **M03.D-M Measurement and Data Reporting Category** | |
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| ASSESSMENT ANCHOR  M03.D-M.4 Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (page 1 of 2) | |
| Overview and vocabulary: Students should recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **attribute, perimeter, plane figure, linear, area, polygon, side length** | |
| **DESCRIPTOR**  **M03.D-M.4.1:**  Find and use the perimeters of plane figures. | **ELIGIBLE CONTENT**  **M03.D-M.4.1.1** Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, exhibiting rectangles with the same perimeter and different areas, and exhibiting rectangles with the same area and different perimeters. Use the same units throughout the problem. |
| **PA:CC.2.4.3.A.6:** Solve problems involving perimeters of polygons and distinguish between linear and area measures.  **Common Core: 3.MD.8**: Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.    **Unpacking**: •Determine the perimeter of any polygon when the side lengths are known;  •Determine any of the side lengths when the perimeter and the other side lengths are known;  •Provide examples of rectangles that have the same perimeter but different areas; and  •Provide examples of rectangles that have the same area but different perimeters.  Students develop an understanding of the concept of perimeter by walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing around a shape on an interactive whiteboard. They find the perimeter of objects; use addition to find perimeters; and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles. Students use geoboards, tiles, and graph paper to find all the possible rectangles that have a given perimeter (e.g., find the rectangles with a perimeter of 14 cm.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Given a perimeter and a length or width, students use objects or pictures to find the missing length or width. They justify and communicate their solutions using words, diagrams, pictures, numbers, and an interactive whiteboard. Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g. find the rectangles that have an area of 12 square units.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.    The patterns in the chart allow the students to identify the factors of 12, connect the results to the commutative property, and discuss the differences in perimeter within the same area. This chart can also be used to investigate rectangles with the same perimeter. It is important to include squares in the investigation. | |

Table 1 Common addition and subtraction situations

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|  | **Result Unknown** | **Change Unknown** | **Start Unknown** |
| **Add to** | Two bunnies sat on the grass. Three  more bunnies hopped there. How many  bunnies are on the grass now?  2 + 3 = ? | Two bunnies were sitting on the grass.  Some more bunnies hopped there. Then  there were five bunnies. How many  bunnies hopped over to the first two?  2 + ? = 5 | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?  ? + 3 = 5 |
| **Take From** | Five apples were on the table. I ate two  apples. How many apples are on the  table now?  5 – 2 = ? | Five apples were on the table. I ate  some apples. Then there were three  apples. How many apples did I eat?  5 – ? = 3 | Some apples were on the table. I ate two  apples. Then there were three apples.  How many apples were on the table  before? ? – 2 = 3 |
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|  | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown2** |
| **Put Together/ Take Apart3** | Three red apples and two green apples  are on the table. How many apples are  on the table?  3 + 2 = ? | Five apples are on the table. Three are  red and the rest are green. How many  apples are green?  3 + ? = 5, 5 – 3 = ? | Grandma has five flowers. How many  can she put in her red vase and how  many in her blue vase?  5 = 0 + 5, 5 = 5 + 0  5 = 1 + 4, 5 = 4 + 1  5 = 2 + 3, 5 = 3 + 2 |
|  |  |  |  |
|  | Difference Unknown | Bigger Unknown | Smaller Unknown |
| **Compare4** | (“How many more?” version):  Lucy has two apples. Julie has five  apples. How many more apples does  Julie have than Lucy?  (“How many fewer?” version):  Lucy has two apples. Julie has five  apples. How many fewer apples does  Lucy have than Julie?  2 + ? = 5, 5 – 2 = ? | (Version with “more”):  Julie has three more apples than Lucy.  Lucy has two apples. How many apples  does Julie have?  (Version with “fewer”):  Lucy has 3 fewer apples than Julie.  Lucy has two apples. How many apples  does Julie have?  2 + 3 = ?, 3 + 2 = ? | (Version with “more”):  Julie has three more apples than Lucy.  Julie has five apples. How many apples  does Lucy have?  (Version with “fewer”):  Lucy has 3 fewer apples than Julie.  Julie has five apples. How many apples  does Lucy have?  5 – 3 = ?, ? + 3 = 5 |

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2These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

3Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

4For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2 Common multiplication and division situations1

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|  | **Unknown Product**  **3 x 6 = ?** | **Group Size Unknown**  **(“How many in each group?” Division)**  **3 x ? = 18 and 18 ÷ 3 = ?** | **Number of Groups Unknown**  **(“How many groups?” Division)**  **? x 6 = 18 and 18 ÷ 6 = ?** |
| **Equal Groups** | There are 3 bags with 6 plums in  each bag. How many plums are  there in all?  *Measurement example.* You need 3  lengths of string, each 6 inches  long. How much string will you  need altogether? | If 18 plums are shared equally into 3 bags,  then how many plums will be in each bag?  *Measurement example.* You have 18 inches  of string, which you will cut into 3 equal  pieces. How long will each piece of string  be? | If 18 plums are to be packed 6 to a  bag, then how many bags are needed?  *Measurement example.* You have 18  inches of string, which you will cut  into pieces that are 6 inches long. How  many pieces of string will you have? |
| **Arrays2 , Area3** | There are 3 rows of apples with 6  apples in each row. How many  apples are there?  *Area example.* What is the area of a  3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows,  how many apples will be in each row?  *Area example.* A rectangle has area 18  square centimeters. If one side is 3 cm long,  how long is a side next to it? | If 18 apples are arranged into equal  rows of 6 apples, how many rows will  there be?  *Area example.* A rectangle has area 18  square centimeters. If one side is 6 cm  long, how long is a side next to it? |
|  |  |  |  |
|  | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown2** |
| **Compare** | A blue hat costs $6. A red hat costs  3 times as much as the blue hat.  How much does the red hat cost?  *Measurement example.* A rubber  band is 6 cm long. How long will  the rubber band be when it is  stretched to be 3 times as long? | A red hat costs $18 and that is 3 times as  much as a blue hat costs. How much does a  blue hat cost?  *Measurement example.* A rubber band is  stretched to be 18 cm long and that is 3  times as long as it was at first. How long  was the rubber band at first? | A red hat costs $18 and a blue hat  costs $6. How many times as much  does the red hat cost as the blue hat?  *Measurement example.* A rubber band  was 6 cm long at first. Now it is  stretched to be 18 cm long. How many  times as long is the rubber band now as  it was at first? |
| **General** | a x b = ? | a x ? = p and p ÷ a = ? | ? x b = p and p ÷ b = ? |

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1The first examples in each cell are examples of discrete things. These are easier or students and should be given before the measurement examples.

2The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in

the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

3Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially

important measurement situations.

Table 3 The properties of operations.

Here *a, b* and *c* stand for arbitrary numbers in a given number system. The properties apply to the rational number system, the real number system, and the complex number system.

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| **Property** | **Example** |
| *Associative property of addition* | (a + b) + c = a + (b + c) |
| *Commutative property of addition* | a + b = b + a |
| *Additive identity property of 0* | a + 0 = 0 + a = a |
| *Associative property of multiplication* | (a x b) x c = a x (b x c) |
| *Commutative property of multiplication* | a x b = b x a |
| *Multiplicative identity property of 1* | a x 1 = 1 x a = a |
| *Distributive property of multiplication over addition* | a x (b + c) = a x b + a x c |